Problem Set 2

Linear Algebra II: Matrix Algebra

Exercise 1: Matrix Multiplication

a.) Two Matrices

Determine whether the following matrices exist, and if so, compute them: AB, $B'A'$ and BA for

$$A = \begin{pmatrix} 0 & 2 \\ 3 & -5 \\ -2 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} -1 & 2 & -3 \\ 4 & -5 & 6 \end{pmatrix}$$

Hint: Be aware of the rules for transposition and matrix operations to take some shortcuts!

b.) Some more Products

Let

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 \\ 2 & -1 \\ -4 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} -5 & 3 \\ 2 & 4 \end{pmatrix}.$$

Determine whether the following matrices exist, and if so, compute them:

$$1.\quad AB \quad 2.\quad BA \quad 3.\quad BA + C \quad 4.\quad AB + C$$

c.) Right-Multiplication of Vectors and Dimensionality

Let A be the matrix as in b.). What $n \in \mathbb{N}$ must we choose so that $x \in \mathbb{R}^n$ can be right-multiplied to A, i.e. as Ax? What about $A'x$?

Exercise 2: Elementary Matrix Operations

Here, we convince ourselves again that the elementary operations really work in the way we introduced them: Consider the matrix

$$A = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}.$$

Define the matrix E so as to

1. interchange rows 2 and 3 (call the matrix E_1),
2. multiply rows 1 and 3 with $\lambda = 5 \neq 0$ (call the matrix E_2),
3. subtract two times row 1 from row 2 (call the matrix E_3).

Multiply out EA for E_3 and check that indeed, the respective operation is performed.

Exercise 3: Determinant, Definiteness and Eigenvalues

a.) Determinant Rules

For the following matrices, compute the determinant using an appropriate rule.

1. $A = \begin{pmatrix} 3 & 8 \\ 2 & -1 \end{pmatrix}$
2. $B = \begin{pmatrix} -3 & 2 & 4 \\ -6 & 5 & 4 \\ 1 & -1 & 0 \end{pmatrix}$
3. $C = \begin{pmatrix} 0 & 0 & 2 \\ 3 & 1 & -1 \\ 2 & 2 & 4 \end{pmatrix}$

Hint: You can test your understanding of the Laplace method by using an appropriate expansion at 3. (of course, the 3×3 rule is still perfectly fine here as well).

b.) Definiteness

Consider the matrix

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 1 \\ -1 & 1 & -1 \end{pmatrix}$$

Is A positive or negative (semi-)definite? What can you say about the invertability of A from this fact?

Exercise 4: Matrix Inversion

For the following matrices, perform a test for invertability (using e.g. the determinant) and, if possible, compute the inverse matrix, using either a shortcut theorem or the Gauss-Jordan method:

1. $A = \begin{pmatrix} 3 & 8 \\ 2 & -1 \end{pmatrix}$

2. $B = \begin{pmatrix} -3 & 2 & 4 \\ -6 & 5 & 4 \\ 1 & -1 & 0 \end{pmatrix}$

3. $C = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Exercise 5: Eigenvalues, Definiteness and Invertability

Let $A = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$.

Determine the eigenvalues of A. What can you say about the definitness of A? Is A invertible? How could you have checked invertability more directly?