Selling Rationed Goods through Associated Markets

An Alternative Approach to the Underpricing Puzzle

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Selling Rationed Goods...

- Monopolist in a rationed product market [Leading example: concert tickets]
  - Demand far exceeds supply ⇒ no more price vs quantity effect trade-off
- Observation of relatively moderate prices

Why are rationed goods often underpriced?

Selling Rationed Goods...

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Why are rationed goods often underpriced?

- Firm is not (solely) profit-maximizing ⇒ additional/secondary objective

Intrinsic Motivation

- Selling preferentially to fans
- Rewarding loyalty / support

Strategic Motivation

- Non-myopic profit maximization
- Reputational Concerns

- Importance of Consumer Selection

...through Auxiliary Markets

morning when tickets went on sale at 11am. Fans who preordered Taylor Swift's latest album, Midnights, from the US singer's official store were given early access, with tickets for London and Edinburgh on sale

Tesla Offers Cybertruck To Long-Term Shareholders: How To Check If You Qualify

HOME TICKETS

Match tickets: are sold exclusively to United Members and there will be multiple releases and opportunties for our members to purchase tickets throughout the season! Specific release dates will be added to individual

It Takes 50 Hours to Earn the Right to Preorder a Switch 2 From Nintendo

...through Auxiliary Markets

- Secondary objective creates incentives for consumer selection
- Use of additional, interrelated markets ⇒ **auxiliary** market
  - Merchandise
  - Stock market
  - Loyalty programs or additional services

⇒ Controlled/Owned by the primary firm

1. *Under which conditions is there a rationale for selection through auxiliary markets?*
2. *Could auxiliary market selection be an answer to the underpricing puzzle?*

Baseline Model

A Uniform Baseline

Consumers

- Continuous mass [normalised to 1]
- Unit demand
- Heterogeneous in two dimensions: WTP w_i and secondary characteristic θ_i

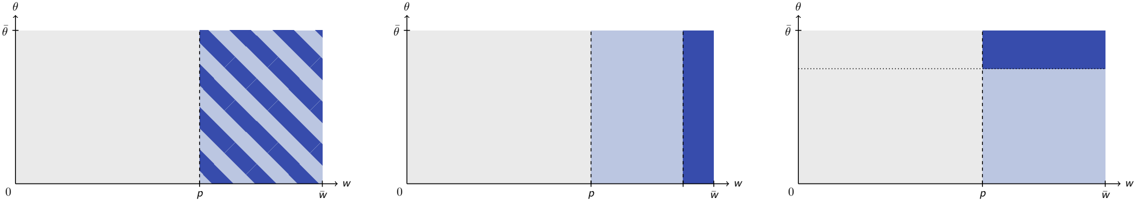
  $\Rightarrow$ joint uniform: $(w_i, \theta_i) \sim U\left([0, \overline{w}] \times [0, \bar{\theta}]\right)$

Monopolist firm

- capacity constraint $k < 1$
- sets uniform price p
- Marginal cost $c = 0$
- Non-standard utility $u^f(p, \theta) = pD(p) + \alpha \int_{\mathcal{C}} \theta_i = D(p)(p + \alpha\Theta)$ $\quad\Theta$: exp. buyers' θ_i

Rationing Rules (aka Allocation Processes)

- Whenever $D(p) > k$, buyers have to be selected from potential consumers.



Proportional rationing

- Default assumption

⇒ First come, first serve

Efficient rationing

- Costly process

⇒ Ownership history

θ-based rationing

- Demanding process

⇒ Lack of advertising

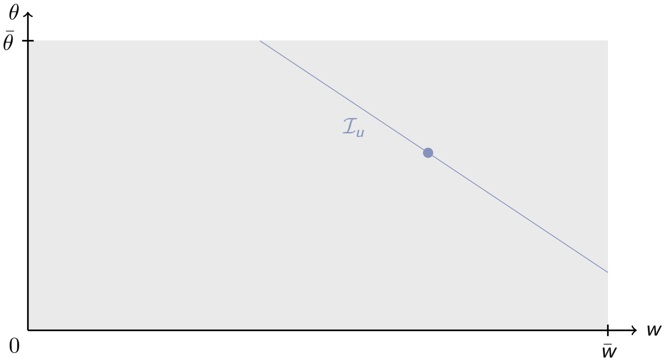
- More complex rationing rules via probability-weighted combinations

Benchmark: Handpicked Consumers

- No uniform price

$\Rightarrow$ Full 1st degr. Discrimination (wrt. both w and θ)

Benchmark: Handpicked Consumers



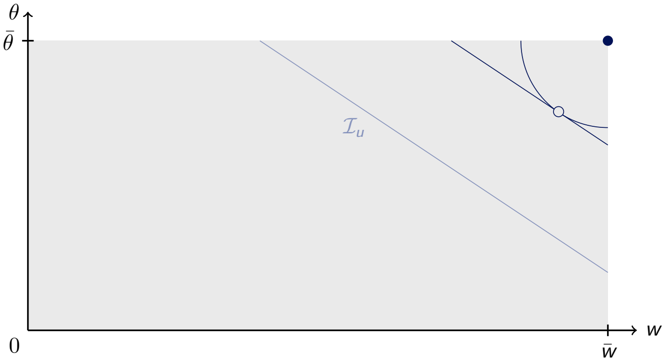
- No uniform price

$\Rightarrow$ Full 1st degr. Discrimination (wrt. both w and θ)

- Select individual consumers

$\Rightarrow$ yields indifference curves $\mathcal{I}_u$

Benchmark: Handpicked Consumers



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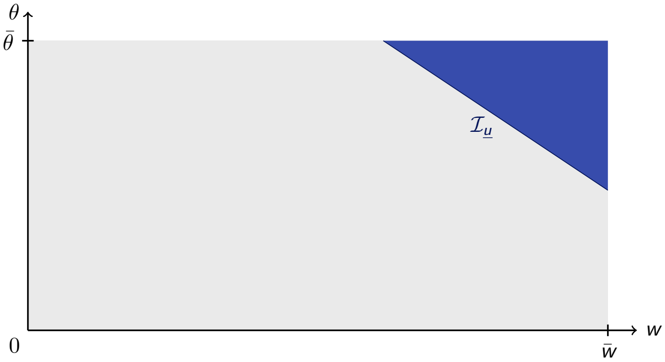


- No uniform price

$\Rightarrow$ Full 1st degr. Discrimination (wrt. both w and θ)

- Select individual consumers

$\Rightarrow$ yields indifference curves $\mathcal{I}_u$

- First-best: $|UCS(\mathcal{I}_u)| = k$

Lemma 1 (First-Best Allocation)

The monopolist's uncontrained optimum is given by serving each $i \in \mathcal{C}^*$ at $p_i = w_i$ where $\mathcal{C}^* := UCS(\mathcal{I}_{\underline{u}})$ for the indifference curve level $\underline{u} := \overline{w} + \alpha\bar{\theta} - \sqrt{2k\overline{w}\bar{\theta}\alpha}$.

Pricing without auxiliary markets

- Back to uniform pricing

  $\Rightarrow$ define clearing price: $\bar{p} := F_w^{-1}(p) = \overline{w}(1-k)$
- Choosing p determines effective demand $D(p)$:

$$D(p) = \begin{cases} k & p \in [0, \bar{p}] \\ \frac{\overline{w}-p}{\overline{w}} & p \in (\bar{p}, \overline{w}] \\ 0 & p \in (\overline{w}, \infty) \end{cases}$$

- Random allocation as default

  $\Rightarrow$ average secondary characteristic $\Theta = E[\theta] = \frac{\bar{\theta}}{2}$ independent of p

p price | k capacity | w WTP | θ secondary characteristic | α non-profit importance

Pricing without auxiliary markets

1. Constrained capacity is fully sold: $D(p) = k$

   $\Rightarrow p < \bar{p}$ dominated by $p^* = \bar{p}$

2. Products are further rationed: $D(p) < k$

   $\Rightarrow \max_p D(p)(p + \alpha\Theta) = \frac{\overline{w}-p}{\overline{w}}\left(p + \alpha\frac{\bar{\theta}}{2}\right) \Rightarrow p^* = \frac{\overline{w}}{2} - \alpha\frac{\bar{\theta}}{4}$

Lemma 2 (Optimal Pricing without Auxiliary Markets)

In the uniform baseline with random allocation, the optimal pricing is given by

$$p^* = \begin{cases} (1-k)\overline{w} & k \leq \frac{2\overline{w}+\alpha\bar{\theta}}{4\overline{w}} \\ \frac{\overline{w}}{2} - \alpha\frac{\bar{\theta}}{4} & k > \frac{2\overline{w}+\alpha\bar{\theta}}{4\overline{w}} \end{cases}$$

For sufficiently small capacity k, the market is fully covered.

Introducing Auxiliary Markets

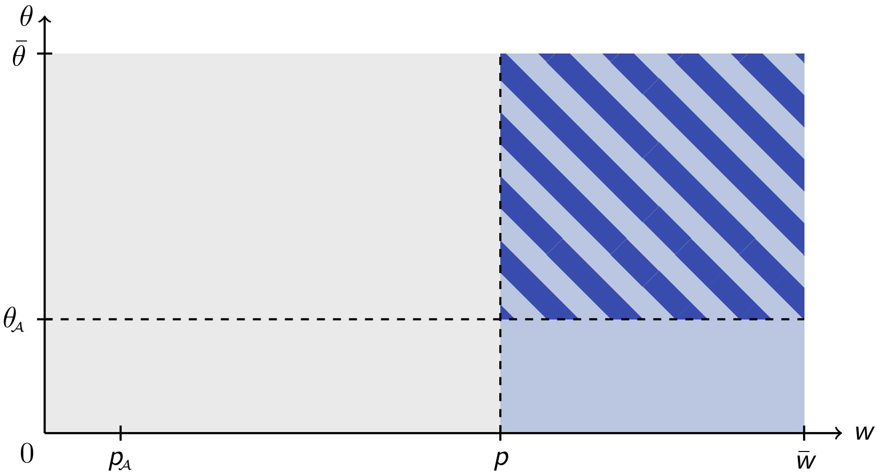
- Secondary, related market $\Rightarrow$ e.g. market for merchandise
- M products of "value" θ_j sold at $p_j = \mu_j \overline{w}$ [μ_j sufficiently small]
  $\Rightarrow$ purchase is observable (or known to the firm) and contractable
- Firm can choose to sell only to consumers who bought any $j \in \mathcal{M}$
- Simplifying assumptions:
  - Identical set of initial consumers
  - Purchase correlated with w_i, θ_i $\Rightarrow$ buy iff $w_i \geq p_j$ and $\theta_i \geq \theta_j$
- The auxiliary markets (incl. M, p_j and θ_j) are exogenous

Observation

Under these assumptions, $|\mathcal{M}| \leq 1$ is weakly optimal.

Optimal Pricing under Auxiliary Market Selection

- Consider any arbitrary auxiliary product with $\theta_{\mathcal{A}}$ (and $p_{\mathcal{A}}$ suff. small)



Optimal Pricing under Auxiliary Market Selection

- Consider any arbitrary auxiliary product with $\theta_{\mathcal{A}}$ (and $p_{\mathcal{A}}$ suff. small)
- Define new clearing price $\bar{p}_{\mathcal{A}}(\theta_{\mathcal{A}}) := \overline{w}\left(1 - k\frac{\bar{\theta}}{\bar{\theta}-\theta_{\mathcal{A}}}\right)$
- As before, effective demand $D_{\mathcal{A}}(p)$ determined by choosing p

$$D_{\mathcal{A}}(p) = \begin{cases} k & p \in [0, \bar{p}_{\mathcal{A}}] \\ \frac{\bar{\theta}-\theta_{\mathcal{A}}}{\bar{\theta}} \frac{\overline{w}-p}{\overline{w}} & p \in (\bar{p}_{\mathcal{A}}, \overline{w}] \\ 0 & p \in (\overline{w}, \infty) \end{cases}$$

- Random allocation together with selection
  $\Rightarrow$ average secondary characteristic $\Theta_{\mathcal{A}} = E[\theta|\theta \geq \theta_{\mathcal{A}}] = \frac{\bar{\theta}+\theta_{\mathcal{A}}}{2}$

Optimal Pricing under Auxiliary Market Selection

1. Constrained capacity is fully sold: $D_{\mathcal{A}}(p) = k$

   $\Rightarrow p < \bar{p}_{\mathcal{A}}$ dominated by $p^* = \bar{p}_{\mathcal{A}}$

2. Products are further rationed: $D_{\mathcal{A}}(p) < k$

   $\Rightarrow \max_p D_{\mathcal{A}}(p)(p + \alpha\Theta_{\mathcal{A}}) = \frac{\bar{\theta}-\theta_{\mathcal{A}}}{\bar{\theta}} \frac{\overline{w}-p}{\overline{w}} \left(p + \alpha \frac{\bar{\theta}+\theta_{\mathcal{A}}}{2}\right) \Rightarrow p^* = \frac{\overline{w}}{2} - \alpha \frac{\bar{\theta}+\theta_{\mathcal{A}}}{4}$

Lemma 3 (Optimal Pricing under Auxiliary Market Selection)

In the uniform baseline, the optimal pricing using the auxiliary market is given by

$$p^* = \begin{cases} \left(1 - k\frac{\bar{\theta}}{\bar{\theta}-\theta_{\mathcal{A}}}\right)\overline{w} & k \leq \frac{\bar{\theta}-\theta_{\mathcal{A}}}{\bar{\theta}} \frac{2\overline{w}+\alpha\bar{\theta}+\alpha\theta_{\mathcal{A}}}{4\overline{w}} \\ \frac{\overline{w}}{2} - \alpha\frac{\bar{\theta}+\theta_{\mathcal{A}}}{4} & k > \frac{\bar{\theta}-\theta_{\mathcal{A}}}{\bar{\theta}} \frac{2\overline{w}+\alpha\bar{\theta}+\alpha\theta_{\mathcal{A}}}{4\overline{w}} \end{cases}$$

For sufficiently small capacity k, the market is fully covered.

Equilibrium of the Uniform Baseline

- When is using the auxiliary market for selection profitable?

  $\Rightarrow$ focus on regular case with sufficiently small k (market covered in both cases)

Illustration: A Decision Problem Inspired Approach

- When is using the auxiliary market for selection profitable?

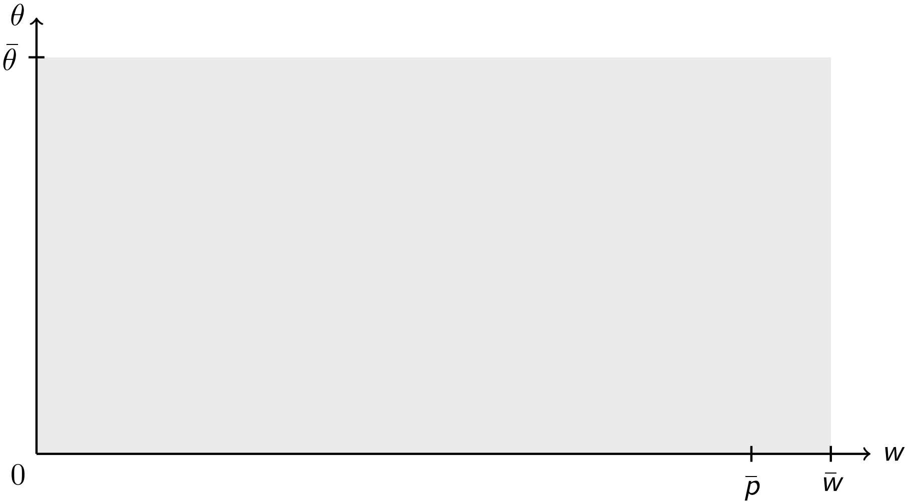


Illustration: A Decision Problem Inspired Approach

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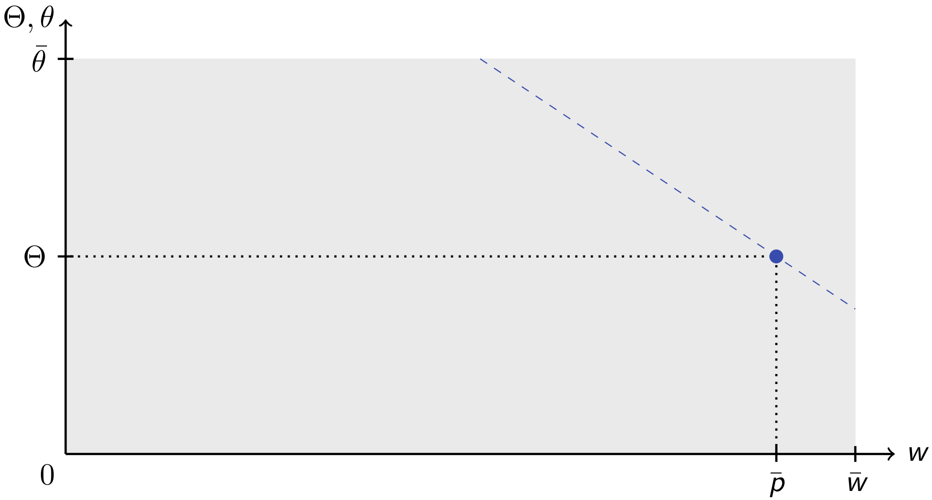

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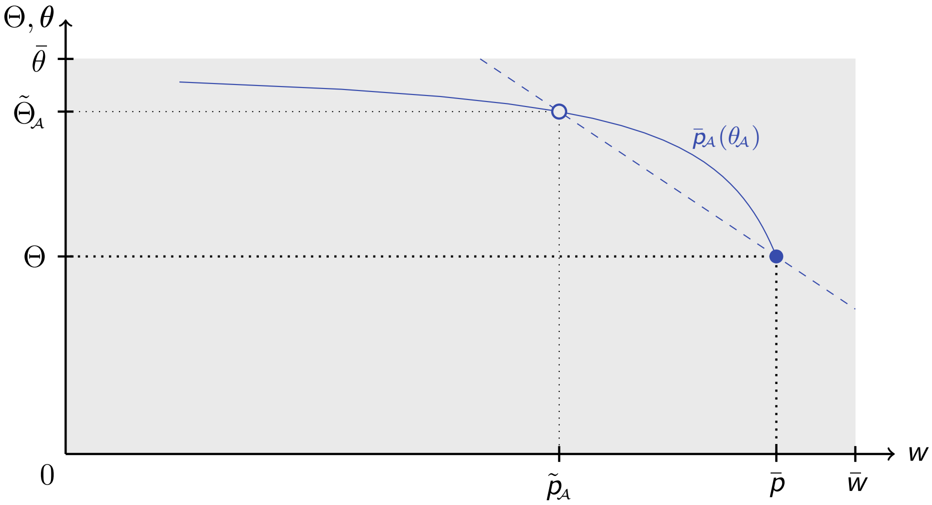

Illustration: A Decision Problem Inspired Approach

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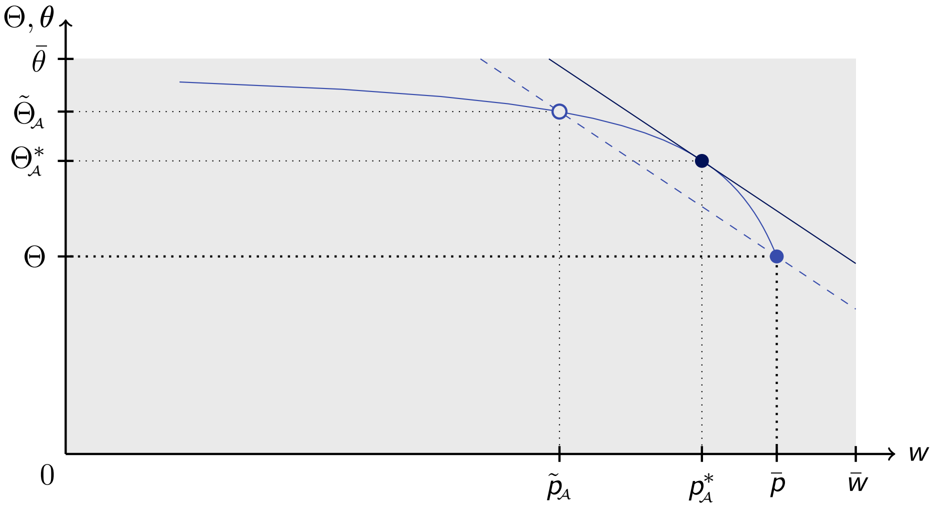


Back to the Equilibrium

- When is using the auxiliary market for selection profitable?

  $\Rightarrow$ focus on regular case with sufficiently small k (market covered in both cases)

  $\Rightarrow$ use auxiliary product whenever $\Theta_{\mathcal{A}} \leq \tilde{\Theta}_{\mathcal{A}} \iff \theta_{\mathcal{A}} \leq \bar{\theta} - \frac{2\overline{w}}{\alpha} k$
- In that case, what is the optimal choice of $\mathcal{M}$?

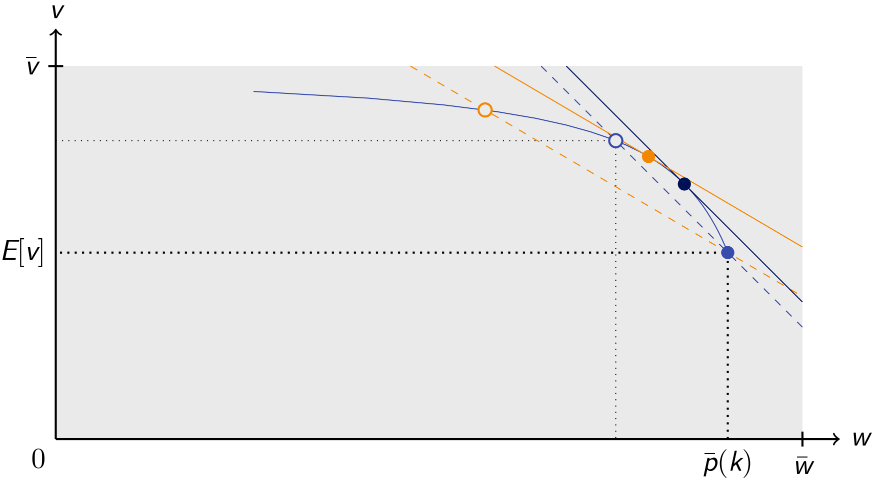
  $\Rightarrow$ single-peaked "preferences": product closest to $\theta^*_{\mathcal{A}} := \bar{\theta} - \sqrt{\frac{2k\overline{w}\bar{\theta}}{\alpha}}$
- If auxiliary market is used: $\bar{p}_{\mathcal{A}} = \left(1 - k\frac{\bar{\theta}}{\bar{\theta}-\theta_{\mathcal{A}}}\right)\overline{w} < (1-k)\overline{w} = \bar{p}$

Proposition 1 (Equilibrium of the Uniform Baseline)

In the uniform baseline, for sufficiently small k, the firm uses the auxiliary market for consumer selection in equilibrium, pricing according to Lemma 3, if there exists a $\theta_{\mathcal{A}} \leq \bar{\theta} - \frac{2\overline{w}}{\alpha} k$ and according to Lemma 2 otherwise.

What about α?

- For sufficiently small k, $\bar{p}_{\mathcal{A}} = \left(1 - k\frac{\bar{\theta}}{\bar{\theta}-\theta_{\mathcal{A}}}\right)\overline{w}$ or $\bar{p} = (1-k)\overline{w}$



Beyond the Simple Baseline

Where to go from here?

Different Allocation Processes

- Allocation based on w or θ
  $\Rightarrow$ analyse mixed/hybrid allocation

Improved Auxiliary Market

- Disentangle consumer base
- Generalize purchase probabilities

Correlated Consumer Type-Space

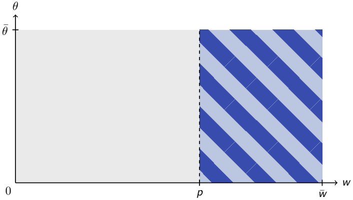
- Dependence of θ on w
  $\Rightarrow$ pricing incentives in baseline

Introduction of Reselling

- Product can be sold on
  $\Rightarrow$ increased need for selection

Different Allocation Processes

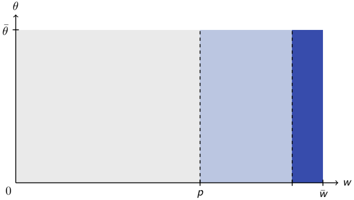
- Whenever $D(p) > k$, buyers have to be selected from potential consumers.



Random allocation

- Default assumption

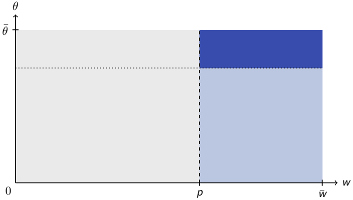
$\Rightarrow$ First come, first serve



w-based allocation

- Costly process

$\Rightarrow$ Ownership history



θ-based allocation

- Demanding process

$\Rightarrow$ Lack of advertising

- More complex allocation processes as probability-weighted combinations

Different Allocation Processes

w-based Allocation

- No pricing below $\bar{p}$ or $\bar{p}_{\mathcal{A}}$ $\Rightarrow$ allocation effectively unchanged

Corollary 1 (Equilibrium under *w*-based Allocation)

The results in Proposition 1 extend to *w*-based Allocation

θ-based Allocation

- Choosing $p < \bar{p}$ selects consumers $\theta \geq \underline{\theta}_p$ $\Rightarrow$ choose $\underline{\theta}_p$ analogously
- As if creating optimal auxiliary market product (at $p_{\mathcal{A}} = 0$)

Corollary 2 (Equilibrium under θ-based Allocation)

Under θ-based Allocation, for sufficiently low k, the optimal p is such that $\underline{\theta}_p = \theta^*_{\mathcal{A}}$.

Different Allocation Processes

Mixed Allocation

- Allocation based on θ with probability λ_θ
- Effective demand $D(p)$ independent of allocation process
- Optimal pricing without auxiliary market [for sufficiently small k]

$$p^* = \begin{cases} (1-k)\overline{w} & \lambda < \bar{\lambda} := \frac{2k\overline{w}}{\alpha\bar{\theta}} \\ \left(1 - \sqrt{\frac{\alpha\lambda\bar{\theta}}{2\overline{w}k}}k\right)\overline{w} & \lambda \geq \bar{\lambda} \end{cases}$$

- Optimal pricing with auxiliary market [for sufficiently small k]

$$p^* = \begin{cases} \left(1 - \frac{\bar{\theta}}{\bar{\theta}-\theta_{\mathcal{A}}}k\right)\overline{w} & \lambda < \bar{\lambda}_{\mathcal{A}} := \frac{2k\overline{w}\bar{\theta}}{\alpha(\bar{\theta}-\theta_{\mathcal{A}})^2} \\ \left(1 - \sqrt{\frac{\alpha\lambda\bar{\theta}}{2\overline{w}k}}k\right)\overline{w} & \lambda \geq \bar{\lambda}_{\mathcal{A}} \end{cases}$$

Different Allocation Processes

Mixed Allocation

- When is using the auxiliary market for selection profitable?
  $\Rightarrow$ focus on regular case with sufficiently small k (market covered in both cases)

1. $\lambda \in (\bar{\lambda}_{\mathcal{A}}, 1)$: only difference $\Theta_{\mathcal{A}} > \Theta$ for $1 - \lambda_\theta$
   $\Rightarrow$ auxiliary product always profitable [but $\bar{\lambda}_{\mathcal{A}}$ increases in $\theta_{\mathcal{A}}$]
2. $\lambda \in (0, \bar{\lambda})$: effectively baseline case
   $\Rightarrow$ use auxiliary product whenever $\theta_{\mathcal{A}} \leq \bar{\theta} - \frac{2\overline{w}}{\alpha} k$
3. $\lambda \in (\bar{\lambda}, \bar{\lambda}_{\mathcal{A}})$: improved baseline ($p^* < \bar{p}$) $\leftrightarrow$ standard auxiliary market ($p^* = \bar{p}_{\mathcal{A}}$)
   $\Rightarrow$ existence of upper threshold for auxiliary product use

Corollary 3 (Equilibrium under Mixed Allocation)

Under mixed Allocation, for sufficiently low k, the firm uses the auxiliary market in equilibrium [...] if there exists a sufficiently low $\theta_{\mathcal{A}}$.

Correlated Consumer Type-Space

- Crucial assumption in the uniform baseline: independence of w and θ

  $\Rightarrow$ introduce (simple, linear) codependence structure: $F_\theta(x|w) = F_\theta(x \pm w|0)$

Positive correlation

Negative correlation

- Incentives to adjust prices in the baseline
- Allocation rule matters [random and w-based distinct]

Pricing under Positive Correlation

- No change in marginal distribution of w

  $\Rightarrow$ clearing price without auxiliary market remains $\bar{p} := F_w^{-1}(p) = \overline{w}(1-k)$

- Effective demand $D(p)$ unchanged as well:

$$D(p) = \begin{cases} k & p \in [0, \bar{p}] \\ \frac{\overline{w}-p}{\overline{w}} & p \in (\bar{p}, \overline{w}] \\ 0 & p \in (\overline{w}, \infty) \end{cases}$$

- Correlation results in Θ increasing with p

  $\Rightarrow$ average secondary characteristic $\Theta(p) = \frac{\bar{\theta}}{2} + \delta \frac{\overline{w}+p}{2}$

p price | k capacity | w WTP | θ secondary characteristic | α non-profit importance

Pricing under Positive Correlation

1. Constrained capacity is fully sold: $D(p) = k$

   $\Rightarrow p < \bar{p}$ dominated by $p^* = \bar{p}$

2. Products are further rationed: $D(p) < k$

   $\Rightarrow \max_p D(p)(p + \alpha\Theta(p)) = \frac{\overline{w}-p}{\overline{w}}\left(p + \alpha\frac{\bar{\theta}}{2} + \alpha\delta\frac{\overline{w}+p}{2}\right) \Rightarrow p^* = \frac{\overline{w}}{2} - \alpha\frac{\bar{\theta}+\delta\overline{w}}{4+2\alpha\delta}$

Lemma 4 (Optimal Pricing under Positive Correlation)

In the uniform baseline with positive correlation, the optimal pricing is given by

$$p^* = \begin{cases} (1-k)\overline{w} & k \leq \frac{2\overline{w}+\alpha\bar{\theta}+2\alpha\delta\overline{w}}{4\overline{w}+2\alpha\delta\overline{w}} \\ \frac{\overline{w}}{2} - \alpha\frac{\bar{\theta}+\delta\overline{w}}{4+2\alpha\delta} & k > \frac{2\overline{w}+\alpha\bar{\theta}+2\alpha\delta\overline{w}}{4\overline{w}+2\alpha\delta\overline{w}} \end{cases}$$

For sufficiently small capacity k, the market is fully covered.

Pricing under Positive Correlation

- Now consider auxiliary product with $\theta_{\mathcal{A}} > \delta\overline{w}$
  $\Rightarrow$ then $\Theta_{\mathcal{A}}(p) = \frac{\bar{\theta}}{2} + \frac{\theta_{\mathcal{A}}}{2} + \delta\frac{\overline{w}+p}{4}$ [otherwise $\Theta_{\mathcal{A}}(p)$ defined differently]
- Effective demand given by

$$D_{\mathcal{A}}(p) = \begin{cases} k & p \in [0, \bar{p}_{\mathcal{A}}] \\ \frac{\bar{\theta}+\delta\frac{\overline{w}+p}{2}-\theta_{\mathcal{A}}}{\bar{\theta}}\frac{\overline{w}-p}{\overline{w}} & p \in (\bar{p}_{\mathcal{A}}, \overline{w}] \\ 0 & p \in (\overline{w}, \infty) \end{cases}$$

- Clearing price $\bar{p}_{\mathcal{A}}$ characterized by $\frac{2\bar{\theta}+\delta\overline{w}+\delta p-2\theta_{\mathcal{A}}}{2\bar{\theta}}\frac{\overline{w}-p}{\overline{w}} = k$

$$\bar{p}_{\mathcal{A}} = \frac{\theta_{\mathcal{A}} - \bar{\theta}}{\delta} + \sqrt{\frac{(\bar{\theta} - \theta_{\mathcal{A}})^2}{\delta^2} + \frac{2\overline{w}(\bar{\theta} - \theta_{\mathcal{A}})}{\delta} + \overline{w}^2 - \frac{2k\overline{w}\bar{\theta}}{\delta}}$$

Where to go from here?

Different Allocation Processes

- Allocation based on w or θ
  $\Rightarrow$ analyse mixed/hybrid allocation

Improved Auxiliary Market

- Disentangle consumer base
- Generalize purchase probabilities

Correlated Consumer Type-Space

- Dependence of θ on w
  $\Rightarrow$ pricing incentives in baseline

Introduction of Reselling

- Product can be sold on
  $\Rightarrow$ increased need for selection

Improved Auxiliary Market

- Disentangle primary and auxiliary market consumer bases

  $\Rightarrow$ products with potentially varying overlap

  $\Rightarrow$ fraction γ_j of primary consumer reached [market share potentially relevant]

- Substantial prices

- Non-binary purchasing probabilities

- Predictability of auxiliary markets or products

Introduction of Reselling

- In practice, measures also aimed at preventing resales

  ⇒ transfer to highest w (irrespective of θ) without extracting rent

- Introduce reselling between existing consumers

  ⇒ likeliness of reselling decreasing in θ_i

- Alternatively: Brokers as new consumers with $\theta_b = 0$

  ⇒ Can be effectively prevented by auxiliary markets

  ⇒ Requires unpredictability of selection criteria

What we've seen so far

- Auxiliary markets effective tool for selecting consumers
  - $\Rightarrow$ correlation between auxiliary product and secondary characteristic
  - $\Rightarrow$ not too niche product
  - $\Rightarrow$ low predictability of market and product choice
- Usage of auxiliary markets can lead to underpricing
  - $\Rightarrow$ allows targeting of specific consumer types
  - $\Rightarrow$ sell to more desirable customers at lower prices
- Choice of auxiliary market might allow reverse engineering of firm's preferences

Thank you for your attention!

Please contact me via mail if you have questions or suggestions or want to receive upcoming versions of this paper.

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