Advertising, Social Media and Parasocial Recommendations

Julian Klix | **Preliminary Draft** **April 2026**

UNIVERSITÄT MANNHEIM

Motivation

WIRED

BOONE ASHWORTH GEAR OCT 12, 2023 8:00 AM

Review: The Air Up Water Bottle

This bottle's smelly gimmick commits a terrible water foul.

Rating: 2/10

WIRED	TIRED
Holds water.	Flavors "taste" funky. Leaks when you turn it on its side. Too expensive. Refilling it is a pain. The straw doesn't work without a disposable scent pod attached. Even then, the straw doesn't really work.

THE AIR UP is a hydration device that smells at you like water bottle is just like flavor water inside.

If you've been on Instagram recently, you may have seen one of the 6 billion sponsored ads for the Air Up. If these ads have made you curious about it, let me save you some time and money: You can easily recreate the Air Up experience by sipping some water while huffing a lemon-scented Glade Plug-In.

Motivation

The Verge

NEWS Updated Sep 15, 2025 at 6:54 PM GMT+2

Honey: all the news about PayPal's alleged scam coupon app

MAY 23
MIA SATO

The Honey exposé fallout continues. Six months after the coupon hunting extension Honey was accused of cheating shoppers and influencers, it appears the PayPal-owned tool is still losing users. According to *9to5Google*, the number of Chrome extension users continues to drop — at one point 20 million people used the extension. Now, that number is down to 15 million.

MAR 12
JAY PETERS

Google changes Chrome extension policies following the Honey link scandal

Google has updated its affiliate ads policy for Chrome extensions after creators accused PayPal's popular Honey browser extension of being a "scam."

Honey was accused of taking affiliate revenue from the same influencers it paid for promotion by using its Chrome extension to swap in its own affiliate link before you checked out.

honey

Honey rose to prominence primarily through an extensive social media marketing campaign. Some of the world's biggest content creators — including *MrBeast* (343 million YouTube subscribers) — produced promotional clips for the browser add-on.

BUSINESS INSIDER

Introduction

Parasocial relationships (PSRs) are nonreciprocal socio-emotional connections that people develop with media figures such as celebrities or fictional characters. PSRs mirror offline social relationships in many ways even though individuals' perceived intimacy with media figures is not reciprocated.

Hoffner, Bond (2022), *Parasocial Relationships, Social media, & Well-being*

- Deviation from traditional advertising $\Rightarrow$ focus on misleading role
- Model of skewable non-strategic recommendations (Benkert, Schmutzler, 2024)

- Monopolist and duopolist with heterogeneous product quality
- Endorsements potentially beneficial under competition, detrimental without

Worse products more likely advertised through social media.
But that's not necessarily bad.

Model Setup

Monopolist Baseline

Model Setup: The Firm Side

The Monopolist

- Single firm with either high (H) or low (L) quality product [generically q]
- Realization of q is private information with $P[q = H] \equiv \theta$
- Produced at constant, homogeneous marginal cost c
- Competes (indirectly) against outside option

The Outside Option

- Available instead of purchasing the product
- Realization of outside option incurs cost c_0

  $\Rightarrow$ Derivable from background Bertrand competition with cost c_0 $\Rightarrow p_0 = c_0$

Model Setup: The Consumer Side

- One (representative) consumer with unit demand
- Receives utility v_0 from background product and v_q from the new product
  $\Rightarrow$ the background product is of intermediate quality, i.e $v_H > v_0 > v_L$

- Can obtain **one** recommendation r prior to purchase
  $\Rightarrow$ generally, positive (r^+) or negative (r^-) possible **[today only positive: $r = r^+$]**
- A product of quality q generates a (positive) recommendation with $P[r|q] \equiv \lambda_q$
  $\Rightarrow$ recommendations are informative, i.e. $\lambda_H > \lambda_L$

Assumption 1

The outside option v_0 is sufficiently large for non-recommended products to not be bought.

An Endorsement-Free Benchmark

- Outside option of $v_0 - c_0$ $\Rightarrow$ by assumption, obtained without recommendation
- Consumer updates belief about q after receiving a recommendation: $\theta \xrightarrow{r} \theta_r$

$$\theta_r \equiv P[H|r] = \frac{P[r|H] \cdot P[H]}{\sum P[r|q] \cdot P[q]} = \frac{\theta \lambda_H}{\theta \lambda_H + (1-\theta)\lambda_L} = \frac{\theta}{\theta + (1-\theta)\frac{\lambda_L}{\lambda_H}} \equiv \Theta(\tfrac{\lambda_L}{\lambda_H})$$

$\Rightarrow$ expected utility $v_r \equiv v_L + \theta_r(v_H - v_L)$ from recommended good

- Firm focuses on customers with recommendation (extracting all surplus)

$$v_r - p = v_0 - c_0 \quad \Longleftrightarrow \quad p^* = v_r - v_0 + c_0 \equiv \Delta v_r + c_0 \equiv p(v_r)$$

Equilibrium of the Advertising-Free Benchmark

Without endorsements, there is a unique equilibrium in which $p^* = \Delta v_r + c$ and consumers buying the new good only if recommended.

Endorsements through Social Media

- Firm can advertise via social media by purchasing "artificial" recommendations
  $\Rightarrow$ inflate recommendation prob. $\lambda_q \rightarrow \hat{\lambda}_q = \lambda_q^g + \lambda_q^f > \lambda_q$ at cost $\kappa^{\mathcal{S}}$
- **Crucial Assumption:** η "naive" consumers (no belief updating)
  $1-\eta$ regular (rational) consumers (distinguish λ_q^g, λ_q^f)
- Correspoding belief $\Theta(\frac{\lambda_L^j}{\lambda_H^j})$ depends on relevant recommendation probability λ_q^j
  $\Rightarrow$ get purchase-inducing recommendation with $\eta\hat{\lambda} + (1-\eta)\lambda^g = \lambda^g + \eta\lambda^f$

- Utilize social media iff $(\lambda_q^g + \eta\lambda_q^f)(\Delta v_r - \Delta c) - \kappa^{\mathcal{S}} \geq \lambda_q(\Delta v_r - \Delta c)$
  $\Rightarrow$ Endorse iff $\kappa^{\mathcal{S}} \leq \bar{\kappa}_q^{\mathcal{S}} \equiv (\lambda_q^g + \eta\lambda_q^f - \lambda_q)(\Delta v_r - \Delta c)$
- Order of $\bar{\kappa}_q^{\mathcal{S}}$ depends on absolute increases $\lambda_q^g + \eta\lambda_q^f - \lambda_q$

Endorsements through Social Media: A Parametrization

- Consider replacing genuine (random) reviews with fabricated ones at rate ρ
  $\Rightarrow$ then $\hat{\lambda}_q = \rho + (1-\rho)\lambda_q$
- Increase is higher for L, relative $(1 - \rho + \eta\rho/\lambda_q)$ and absolute $(\rho(\eta - \lambda_q))$
- Consequently, $\bar{\kappa}_q^{\mathcal{S}}(\rho) = (\eta - \lambda_q)\rho(\Delta v_r - \Delta c) \quad \Longrightarrow \quad \bar{\kappa}_L^{\mathcal{S}}(\rho) > \bar{\kappa}_H^{\mathcal{S}}(\rho)$

Equilibria under Social Media Endorsement (Parametrized)

Under social media advertising, parametrized by $\hat{\lambda}_q = \rho + (1-\rho)\lambda_q$, three classes of equilibria with $p^* = \Delta v_r + c_0$ are possible:

1. $\kappa^{\mathcal{S}} \in [0, \bar{\kappa}_H^{\mathcal{S}}(\rho)]$ Both firm types advertise via social media
2. $\kappa^{\mathcal{S}} \in [\bar{\kappa}_H^{\mathcal{S}}(\rho), \bar{\kappa}_L^{\mathcal{S}}(\rho)]$ Only the L-firm advertises
3. $\kappa^{\mathcal{S}} \in (\bar{\kappa}_L^{\mathcal{S}}(\rho), \infty)$ No firm type advertise via social media

Advertising through Social Media: Generalized Equilibrium

- The three-equilibrium structure generalizes to all updating regimes $\hat{\lambda}$

  $\Rightarrow$ aysmetric case governed by the higher absolute increase $(\eta\lambda_q^f + \lambda_q^g - \lambda_q)$

Equilibria under Social Media Endorsement

Let $q_< \equiv \arg\min_q \eta\,\lambda_q^f + \lambda_q^g - \lambda_q$ and, conversely, $q_>$. Then, three classes of equilibria with $p^* = \Delta v_r - \Delta c$ are possible under social media endorsement:

1. $\kappa^{\mathcal{S}} \in [0, \bar{\kappa}^{\mathcal{S}}_{q_<}]$ Both firm types advertise via social media
2. $\kappa^{\mathcal{S}} \in [\bar{\kappa}^{\mathcal{S}}_{q_<}, \bar{\kappa}^{\mathcal{S}}_{q_>}]$ Only the $q_>$-firm advertises
3. $\kappa^{\mathcal{S}} \in (\bar{\kappa}^{\mathcal{S}}_{q_>}, \infty)$ No firm type advertise via social media

- Low quality firm more likely to use social media when (absolute) gains are higher
- Endorsement likelihood increasing in η and ρ

But What About the Influencers...?

- So far $\kappa^{\mathcal{S}}$ exogenously given and constant
  $\Rightarrow$ derive a more sophisticated $\kappa^{\mathcal{S}}$ based on influencer's decision problem
- Post content each period $t \Rightarrow$ earn profit based on follower engagement ε_j
- Endorsement reduces ε_j if found out (with prob. $\mathbb{P}_j^e$) and costs $k(\rho)$
- Per-consumer perspective, accept endorsement iff

$$\Pi_{\mathcal{S}} - k(\rho) + \sum_{t=0}^{\infty} \delta^t (\eta(\varepsilon_N - \mathbb{P}_N^e \Delta\varepsilon_N) + (1-\eta)(\varepsilon_R - \mathbb{P}_R^e \Delta\varepsilon_R)) \geq \sum_{t=0}^{\infty} \delta^t (\eta\varepsilon_N + (1-\eta)\varepsilon_R)$$

- Without (additional) frictions, $\Pi_{\mathcal{S}} = \kappa^{\mathcal{S}} \Rightarrow$ Accept iff $\kappa^{\mathcal{S}} \geq k(\rho) + \eta[\underbrace{\frac{\delta}{1-\delta}\Delta\varepsilon_N}_{\equiv k_N}]\mathbb{P}_N^e$
- Endorsement cost: $\bar{\kappa}^{\mathcal{S}} = k(\rho) + (1-\theta)\eta k_N \lambda_L^f$
  $\Rightarrow$ if quality observable (to influencer): $\bar{\kappa}_H^{\mathcal{S}} = k(\rho)$ and $\bar{\kappa}_L^{\mathcal{S}} = k(\rho) + \eta k_N \lambda_L^f$

...and About Welfare?

- Denote expected probabilities $\lambda_\mu^j \equiv \theta\lambda_H^j + (1-\theta)\lambda_L^j$
- Welfare & CS in Benchmark: $W_\varnothing = v_0 - c_0 + \lambda_\mu \Delta v_r$ and $CS_\varnothing = v_0 - c_0$
- Welfare & CS with endorsement depends on advertising types:

$$
\begin{aligned}
W_{HL} &= v_0 - c_0 + \lambda_\mu^g \Delta v_r + \eta\rho\Delta v_\mu \\
CS_{HL} &= v_0 - c - \eta\rho(v_r - v_\mu) \\
W_L &= v_0 - c_0 + \lambda_\mu \Delta v_r + (1-\theta)(\eta - \lambda_L)\rho\Delta v_L \\
CS_L &= v_0 - c + \theta\rho\lambda_H(v_H - v_r) + (1-\theta)\eta\rho(v_L - v_\mu) \\
W_H &= v_0 - c_0 + \lambda_\mu \Delta v_r + +\theta(\eta - \lambda_H)\rho\Delta v_H \\
CS_H &= v_0 - c + \theta\eta\rho(v_H - v_\mu) + (1-\theta)\rho\lambda_L(v_L - v_\mu)
\end{aligned}
$$

- One can show $W_H > W_\varnothing > W_{HL} > W_L$ as well as $CS_H > CS_\varnothing > CS_{HL}, CS_L$

Policy Implications

- Most parameters (θ, λ, v) out of scope
- Fraction of parasocial relationships η
  - Reducing η increases CS if L advertises
  - Welfare increasing for HL-endorsement only if $v_\mu < v_0$
  - Reducing η too much may cause equilibrium switch $CS_{HL} \to CS_L$
- Cost of endorsement $k(\rho)$
  - Increase advertising threshold $\bar{\kappa}^S$ $\to$ drive firms out of advertising
  - No direct CS-effect (but potentially adverse equilibrium switch)
- Cost of misleading advertisement k_N
  - Significant difference only with identifiable qualities $\to$ target L specifically
- Social media ban for children/teenager ($\eta \downarrow$) $\longleftrightarrow$ Parental controls ($k_N \uparrow$)

Extensions

Beyond the Monopolist Baseline

Costly Quality Assessment

- Product quality q can be assessed by influencer at cost k_Q

  $\Rightarrow$ previous regimes included as $k_Q = \infty$ and $k_Q = 0$

Equilibrium with Quality Assessment

In the model with costly assessment, there exists a mixed strategy equilibrium (unique up to $\underline{\kappa}$) for sufficiently low k_Q in which

- The H-type offers $\underline{\kappa}$
- The L-type offers $\underline{\kappa}$ with $\sigma_L \equiv \frac{\theta k_Q}{(1-\theta)[\eta k_N \lambda_L^f - \underline{\kappa} + k(\rho) - k_Q]}$ and $\bar{\kappa} = k(\rho) + \eta k_N \lambda_L^f$ otherwise.
- The influencer accepts any $\bar{\kappa}$-offer and gets certification of $\underline{\kappa}$-offers with $\sigma_I \equiv \frac{\bar{\kappa} - \underline{\kappa}}{\bar{\kappa}_S^L - \underline{\kappa}}$

$\Rightarrow$ Assessment only via spot tests $\rightarrow$ some L products slip through

Endogenizing Endorsement Reach

- Allow each quality type to choose an individual ρ_q
  - $\Rightarrow$ Trade-off: profit (via $\hat{\lambda}_q \uparrow$) $\longleftrightarrow$ cost $\kappa_q^{\mathcal{S}}(\rho)$
  - $\Rightarrow$ Crucial role of $k(\rho)$ $\rightarrow$ assume $k' , k'' > 0$
- Each q-type solves $\max_\rho (\eta - \lambda_q)\rho(\Delta v_r - \Delta c) - k(\rho) - (1-\theta)\eta\rho k_N$
  - $\Rightarrow$ Interior optimality condition $k'(\rho) = (\eta - \lambda_q)(\Delta v_r - \Delta c) - (1-\theta)\eta k_N$
  - $\Rightarrow$ convexity & $\lambda_H > \lambda_L$ implies $\rho_L > \rho_H$
- Low quality products are marketed more aggressively
- Extends to identifiable quality ($\kappa_H^{\mathcal{S}} \neq \kappa_L^{\mathcal{S}}$) as long as k_N not too large
- Requires bound on $\lambda_H - \lambda_L$ to prevent unravelling

Endorsement Bandwaggoning

- Alternative approach to adjust ρ: **simultaneous endorsements**
- Individual recommendations with fixed ρ can be aggregated
  $\Rightarrow$ cost is additive, effective reach given by aggregator $a(\rho, i)$
  $\Rightarrow$ $a(\cdot)$ increasing, concave in number of influencer i [e.g. $a(\rho, i) = 1 - (1-\rho)^i$]
- Blame is shared between all influencers
- Firm solves $\max_i (\eta - \lambda_q) a(\rho, i)(\Delta v_r - \Delta c) - i[k(\rho) + (1-\theta)\eta k_N \frac{\lambda_q^f}{i}]$
  $\Rightarrow$ optimality condition $a_i(\rho, i) = \frac{k(\rho)}{(\eta-\lambda_q)(\Delta v_r - \Delta c) - (1-\theta)\eta k_N}$
- Can be mapped one-to-one to the previous problem $\Rightarrow$ $i_L > i_R$
- Endorsements for low quality products appear in larger clusters

Stochastic Value and the Golden Sample [WIP]

- Product value $v_q \in \{\bar{v}, \underline{v}\}$ with $\mathbb{E}v_q = \bar{v} - \psi_q(\bar{v} - \underline{v})$
  $\Rightarrow$ all previous results remain qualitatively unchanged

- Quality assessment based on realization of v_q
  $\Rightarrow$ provide cherry picked sample with acceptance probability γ at cost $c_q(\gamma)$

- **Crucial Assumption:** Influencer unaware of bias ($\Rightarrow \gamma_q$ independent)

- Firm solves $\max_\gamma \gamma[(\lambda_q^g + \eta\lambda_q^f)(\Delta v_r - \Delta c) - \kappa_S] + (1-\gamma)\lambda_q(\Delta v_r - \Delta c) - c_q(\gamma)$
  $\Rightarrow$ optimality condition $c_q'(\gamma) = \rho(\eta - \lambda_q)(\Delta v_r - \Delta c) - \kappa_S$

- If $c_q(\gamma)$ increasing, convex and sufficiently close: $\gamma_L > \gamma_H$
  $\Rightarrow$ potential role of market size ($c_q(\gamma)$ is per-consumer cost)

A Detour to (Standard) Informative Advertising

- Reveal v_q to consumer at cost $\kappa^{\mathcal{A}}$ (available with prob. α)
  $\Rightarrow$ Only profitable for H-firm: charge $p = v_H - v_0 + c = \Delta v_H + c$
- Updated expected utility (L and non-advertising H): $v_{\mathcal{A}} \equiv v_L + \Theta(\frac{\lambda_L}{(1-\alpha)\lambda_H})(v_H - v_L)$
- Endorsement profitable iff $\kappa^{\mathcal{S}} \leq (\eta - \lambda_q)\rho(\Delta v_{\mathcal{A}} - \Delta c)$
- Additional advertising (only H-type) optimal iff
  $$\kappa^{\mathcal{A}} - k(\rho) \leq \Delta v_H - (\eta\rho - \lambda_H\rho + \lambda_H)\Delta v_{\mathcal{A}} - (1 - \eta\rho + \lambda_H\rho + \lambda_H)\Delta c > 0$$
- Advertising channel reduces endorsement likelihood
  $\Rightarrow$ but exacerbates H-L asymmetry

Competitive Model

Endorsement in a Duopoly

A Model with Competing Firms

- Outside option as before $\rightarrow$ $v_0 - c_0$

- Two symmetrical competing firms
  $\Rightarrow$ two independent products with $q_i \in \{H, L\}$ for $i = 1, 2$
  $\Rightarrow$ costs are identical $c_1 = c_2 = c$

- Consumer gets one recommendation each
  $\Rightarrow$ realizations are independent with $\lambda_1 = \lambda_2$
- Given equal expected utilities, consumer is indifferent between firms.

- For simplicity, assume $c_0 = c$

The Benchmark Equilibrium under Competition

- Varian-style competition structure
- Each recommended firm faces a consumer with anonther one with prob. π

  $\Rightarrow$ in baseline: $\pi = \theta\lambda_H + (1-\theta)\lambda_L$

- Both firms play a symmetric mixed strategy $p \sim F$ on $[\underline{p}, \Delta v_r]$

  $\Rightarrow$ Pricing as monopolist: $\Pi(\Delta v_r + c) = (1-\pi)\Delta v_r$

  $\Rightarrow$ Pricing at arbitrary p: $\Pi(p) = (1-\pi)(p-c) + \pi(1-F(p))(p-c)$

- Indifference at all p : $(1-\pi)\Delta v_r = (1-\pi)(p-c) + \pi(1-F(p))(p-c)$

  $\Rightarrow \quad F(p) = \frac{1}{\pi} - \frac{1-\pi}{\pi}\frac{\Delta v_r}{p-c}$ with $\underline{p} = (1-\pi)\Delta v_r + c$

Social Media Advertising under Competition

- Advertising increases λ_i independently (and symmetrical) for both firms
- Derivation as before (with $\hat{\pi}$) yields $\hat{F}(p) = \frac{1}{\hat{\pi}} - \frac{1-\hat{\pi}}{\hat{\pi}} \frac{\Delta v_r}{p-c}$ with $\underline{p} = (1-\hat{\pi})\Delta v_r + c$
- By indifference, $\mathbb{E}\Pi(\cdot) = \Pi(\Delta v_r + c) = (1-\hat{\pi})\Delta v_r$

- Advertise iff $(\lambda_q^g + \eta\lambda_q^f)(1-\hat{\pi})\Delta v_r - \kappa^{\mathcal{S}} \geq \lambda_q(1-\hat{\pi})\Delta v_r$
$$\iff \kappa^{\mathcal{S}} \leq \bar{\kappa}_q^S \equiv (1-\hat{\pi})(\eta - \lambda_q)\rho\Delta v_r$$
$\Rightarrow$ three equilibrium classes, intermediate governed by higher absolute increase

- Equilibrium constellation depends on $\hat{\pi} \in \{\pi_{HL}, \pi_L, \pi_\emptyset\}$ [with $\pi_{HL} > \pi_L > \pi_\emptyset$]
  - HL–HL equilibrium if $\kappa^{\mathcal{S}} \leq (1-\pi_{HL})(\eta - \lambda_H)\rho\Delta v_r$
  - HL–$\emptyset$ eqm. if $\kappa^{\mathcal{S}} \leq (1-\pi_\emptyset)(\eta - \lambda_H)\rho\Delta v_r$ but $\kappa^{\mathcal{S}} > (1-\pi_{HL})(\eta - \lambda_L)\rho\Delta v_r$
  - L–L eqm. if $\kappa^{\mathcal{S}} \leq (1-\pi_L)(\eta - \lambda_L)\rho\Delta v_r$ but $\kappa^{\mathcal{S}} > (1-\pi_L)(\eta - \lambda_H)\rho\Delta v_r$

Consumer Surplus under Competition

- Consumer surplus dependent on expected price of single recommendation firm

$$E[p] = \int_{(1-\pi)\Delta v_r + c}^{\Delta v_r + c} p \frac{1-\pi}{\pi} \frac{\Delta v_r}{(p-c)^2} dp = \frac{1-\pi}{\pi} \ln\left(\frac{1}{1-\pi}\right) \Delta v_r + c$$

- Expected price of double recommendation ($= \min p_j$)

$$E[p] = \int_{(1-\pi)\Delta v_r + c}^{\Delta v_r + c} p \cdot 2\left(\frac{1-\pi}{\pi}\right)^2 \frac{\Delta v_r (p-c-\Delta v_r)}{(p-c)^3} dp = \frac{1-\pi}{\pi}\left[2 - 2\frac{1-\pi}{\pi} \ln\left(\frac{1}{1-\pi}\right)\right] \Delta v_r + c$$

- Consumer surplus higher than under monopoly baseline
- Effect of social media endorsements ambiguous
  - Higher recommendation probability ($\pi_{HL} > \pi_L > \pi_\emptyset$) reduces prices
  - Fabricated reviews increase likelihood of $v_L < v_0$ realizations

Summary

- Parasocial nature of social media can be used strategically
  $\Rightarrow$ confound genuine, informative recommendations with artificial ones
- Worse products more prone to be advertised via social media
- Low quality products tend to be marketed more aggresively and clustered
- Influencer's don't investigate quality thoroughly
- Misleading marketing generally welfare-detrimental
  $\Rightarrow$ can still be beneficial to consumers by increasing competition
- Policy dependent on staus quo
  $\Rightarrow$ excessive intervention can exacerbate problem

Thank you for your attention!

Please contact me via mail if you have questions or suggestions or want to receive upcoming versions of this paper.

julian.klix[at]uni-mannheim.de

jklix

UNIVERSITÄT MANNHEIM