Advertising, Social Media and Parasocial Recommendations

New Infuencer-Centered Modeling Approach

Julian Klix | **IO PhD Seminar** October 21, 2026



Motivation

Parasocial relationships (PSRs) are nonreciprocal socio-emotional connections that people develop with media figures such as celebrities or fictional characters. PSRs mirror offline social relationships in many ways even though individuals' perceived intimacy with media figures is not reciprocated.

Hoffner, Bond (2022), *Parasocial Relationships, Social media, & Well-being*

- Deviation from traditional advertising $\Rightarrow$ focus on deceptive role
- Model of skewable non-strategic recommendations (Benkert, Schmutzler, 2024)

- Monopolist with heterogeneous product quality
- Influencer with other-regarding preferences

Worse products more likely advertised through social media.

But that's not necessarily bad.

Model Setup: The Product Landscape

- Multiple quality dimensions
  - $\Rightarrow$ denote $j \in \{1, 2\}$
  - $\Rightarrow$ relevant for realized utility
  - $\Rightarrow$ Allows for horizontal and vertical differentiation

- Multiple product categories
  - $\Rightarrow$ denote $k \in \{S, G\}$
  - $\Rightarrow$ will allow for influencer "specialization"

  Not used in Baseline model

Model Setup: The Firm Side

The Monopolist

- Single firm with high (h) or low (l) quality in each dimension [generically q_j]
- Realization of all q_j are private information with $P[q_j = H] \equiv \pi$
- Three overall quality classes q: H (both $q_j = h$), M (one $q_j = h$) or L (no $q_j = h$)
- Produced at constant, homogeneous marginal cost c
- Competes (indirectly) against outside option
- Can advertise product on social media at exogenous cost κ

The Outside Option

- Available instead of purchasing the product

  $\Rightarrow$ normalized to $p_0 = 0$

Model Setup: The Consumer Side

- One (representative) consumer with unit demand
- Randomly drawn product dimension type $\theta \in J$
- Receives utility v_0 from outside option and v_q from the new product
  $\Rightarrow$ utility of product given by $u_c(\theta) = v_{q_\theta}$ [i.e. quality of dimension θ]
  $\Rightarrow$ normalize $v_l = 0$, simplify notation $v_h = v$

- Can obtain **one** recommendation r prior to purchase
  $\Rightarrow$ either from environment (non-strategic) or social media (potent. strategic)

Assumption 1

The outside option v_0 is sufficiently large to dominate random purchase.
That is, $v_0 > \pi v - p$.

Model Setup: The Influencer Side

- One (representative) Influencer that can give recommendations
  - Product dimension type $\iota \in J$
  - direct utility from payment $\Pi_{\mathcal{I}}$, indirect utility from value-added to consumer
    $\Rightarrow$ altruism parameter α captures trade-off, distr. following (cont.) F_α

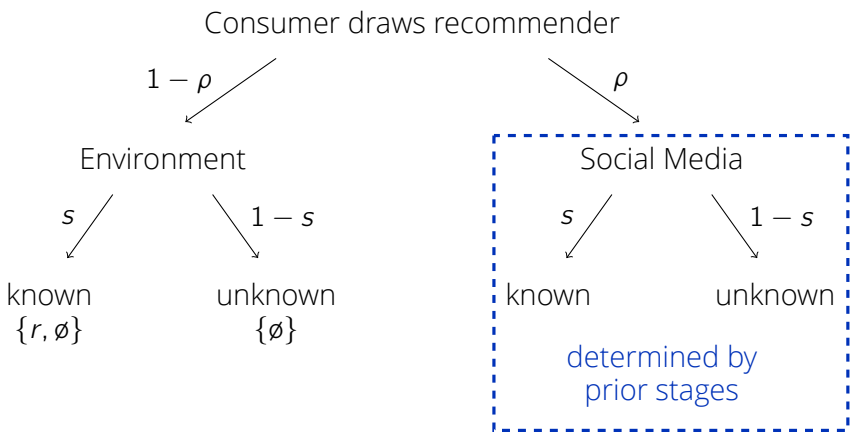
    $$u_{\mathcal{I}} = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[u_c(r) - u_c(\emptyset)] = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[\mathbb{E}[v_\theta|r] - p - v_0]$$

  $\Rightarrow$ non-strategic environment encapsuled by $\alpha = 1$
- Type-match probability ("similarity") σ $\Rightarrow$ potentially σ_0, σ_I

Parasocial Relationships

The consumer's parasocial relationship is captured by overestimating the influencer's similarity, i.e., $\sigma_\eta > \sigma$. [Specific form to be determined]

Model Setup: The Recommendation Procedure



Model Setup: Timing

1. Firms learn q, knowing recommenders learn v_ι
2. Firm decides on endorsement offer
3. Influencer decides on accepting endorsement offer
4. Consumer draws recommender
5. Consumer observers potential recommendation
   $\Rightarrow$ can distinguish genuine from paid recommendations
6. Consumer makes purchase decision

$\Rightarrow$ utilities are realized

An Endorsement-Free Benchmark

- Outside option of v_0 $\Rightarrow$ by assumption, obtained without recommendation
- Consumer updates belief about v_θ after receiving a recommendation: $\pi \xrightarrow{r} \pi_r$

$$\pi_r \equiv \mathbb{P}[v_\theta = v|r] = \sigma \frac{1 \cdot \mathbb{P}[v]}{1 \cdot \mathbb{P}[v] + 0 \cdot \mathbb{P}[0]} + (1-\sigma) \frac{\pi \cdot \mathbb{P}[v]}{\pi \cdot \mathbb{P}[v] + \pi \cdot \mathbb{P}[0]} = \sigma + (1-\sigma)\pi$$

$\Rightarrow$ assume expected utility $\pi_r v - p = \pi v + (1-\pi)\sigma v - p > v_0$

- without endorsements ($\Pi_{\mathcal{I}} = 0$), influencer's interests perfectly aligned

  $\Rightarrow$ recommend a (known) product iff $v_\iota = v$

Equilibrium of the Endorsement-Free Benchmark

Without endorsements, there exists an equilibrium in which any high-value product gets recommended and all recommendations are followed.

The HML-Endorsement Equilibrium

HML-Endorsement Equilibrium

For sufficiently small κ there exists an equilibrium with endorsements in which

(a) All firm types ($q = H, M, L$) decide to seek endorsements at cost κ.

(b) There exist thresholds $\bar{\alpha}_s$ and $\bar{\alpha}_{\bar{s}}$ such that an influencer

  i. with $\alpha \leq \bar{\alpha}_{\bar{s}}$ endorses the product unseen.

  ii. with $\alpha \leq \bar{\alpha}_s$ endorses any product and $\alpha > \bar{\alpha}_s$ only $v_\iota = v$ products.

(c) All consumers follow genuine recommendations.

(d) Sufficiently parasocial consumers ($\sigma_\eta > \bar{\sigma}$) also follow endorsements.

- Equilibrium with full firm-type pooling

Firm Decision

- Firm endorses if costs are sufficiently small $\kappa \leq (D_E^q - D_0^q)(p - c)$

  $\Rightarrow$ effective demand with (D_E^q) and without (D_0^q) endorsements depends on v_ι

- Denote share of parasocial customers ($\sigma_\eta > \bar{\sigma}$) by ψ_η

v_ι	D_0	D_E
V	s	$\rho(1-s)\psi_\eta F_\alpha(\bar{\alpha}_{\bar{s}}) - \rho s(1-\psi_\eta) + s$
0	0	$\rho(1-s)\psi_\eta F_\alpha(\bar{\alpha}_{\bar{s}}) + \rho s\psi_\eta F_\alpha(\bar{\alpha}_s)$

- Note that absolute gains ($D_E^q - D_0^q$) decrease with q_ι (and hence with q)

  $\Rightarrow$ all firms profit iff $\kappa \leq \bar{\kappa}_H \equiv \rho[\psi_\eta(1-s)F_\alpha(\bar{\alpha}_{\bar{s}}) - (1-\psi_\eta)s](p-c)$

  $\Rightarrow$ Note: $\bar{\kappa}_H > 0 \iff \frac{\psi_\eta}{1-\psi_\eta} F_\alpha(\bar{\alpha}_{\bar{s}}) > \frac{s}{1-s}$ $\quad \rightarrow$ necessary cond. for HML-eq.

Consumer Decision

- Reminder: Consumers can differentiate paid from genuine recommendations
  $\Rightarrow$ paid ones can still be **aligned** (i.e., iff $v_\iota = v$) for high α $\Rightarrow$ denote prob. ψ_α

- Genuine recommendations always aligned (Benchmark) $\Rightarrow$ always followed

- Follow paid recommendation iff v_0 or p sufficiently small and

  $$\mathbb{E}[v_\theta | r^p] = \psi_\alpha(\pi + (1-\pi)\sigma)v + (1-\psi_\alpha)\pi v = [\pi + (1-\pi)\sigma\psi_\alpha]v \geq v_0 + p$$

  $\Rightarrow$ satisfied for sufficiently parasocial consumers $\Rightarrow \sigma_\eta \geq \bar{\sigma} \equiv \frac{v_0+p}{(1-\pi)\psi_\alpha v} - \frac{\pi}{(1-\pi)\psi_\alpha}$

  $\Rightarrow$ denote probability of following paid recommendations by ψ_η

Influencer Decision

- Recall influencer's utility: $u_{\mathcal{I}} = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[u_c(r) - v_0]$
- Three influencer types, accept endorsement iff
  - $\bar{s}: \quad (1-\alpha)\kappa + \alpha[\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \pi v + p + v_0}$
  - $s|v: \ (1-\alpha)\kappa + \alpha[(1-\pi)\sigma v + \pi v - p - v_0] > (1-\alpha)\kappa \geq 0$
    $\Rightarrow$ always true (interests perfectly aligned)
  - $s|0: (1-\alpha)\kappa + \alpha[(1-\sigma)\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_s \equiv \frac{\kappa}{\kappa - (1-\sigma)\pi v + p + v_0}$

$\Rightarrow$ aligned paid recommendation probability $\psi_\alpha = \frac{\pi s(1-F_\alpha(\bar{\alpha}_s))}{s(\pi+(1-\pi)F_\alpha(\bar{\alpha}_s))+(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})}$

Comparative Statics

Influencer

- $\bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \pi v + p + v_0}$ $\quad$ $\bar{\alpha}_s \equiv \frac{\kappa}{\kappa - (1-\sigma)\pi v + p + v_0}$

  $\Rightarrow$ incr. in κ, π, v $\quad$ decr. in p, v_0 $\quad$ incr. in κ, π, v $\quad$ decr. in p, v_0, σ

- $\psi_\alpha \equiv \frac{\pi s(1-F_\alpha(\bar{\alpha}_s))}{s(\pi+(1-\pi)F_\alpha(\bar{\alpha}_s))+(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})}$

  $\Rightarrow$ incr. in s, p, v_0, σ $\quad$ decr. in κ, v and $\bar{\alpha}$ $\quad$ ambiguous in π

Consumer

- $\bar{\sigma} \equiv \frac{v_0 + p - \pi v}{(1-\pi) v \psi_\alpha}$

  $\Rightarrow$ incr. in κ $\quad$ decr. in ψ_α, s, σ

Firm

- $\bar{\kappa}_H \equiv \rho[\psi_\eta(1-s)F_\alpha(\bar{\alpha}_{\bar{s}}) - (1-\psi_\eta)s](p-c)$

  $\Rightarrow$ incr. in ρ $\quad$ decr. in c

Consumer Surplus & Welfare

- Welfare as aggregated utility (omitting other-regarding preferences)

  $W = \mathbb{E}_{\text{HML}} v - p + p - c - \kappa + (1-\alpha)\kappa + \alpha(E[v|r] - p - v_0)$

  $\Rightarrow$ p, κ welfare neutral $\rightarrow$ weighting factor for influencer utility

  $W = \mathbb{E}_{\text{HML}} v - p + p - c - \kappa + \frac{1}{1-\alpha}(1-\alpha)\kappa = \mathbb{E}_{\text{HML}} v - c = CS + p - c$

- Focus on (net) consumer surplus $CS - v_0$ $\Rightarrow$ multiple components

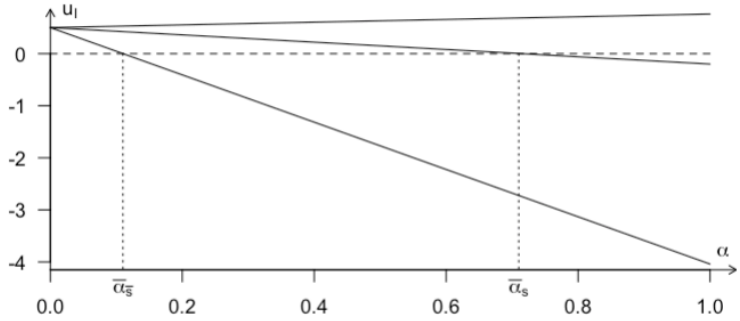
r_g:	$[\sigma + (1-\sigma)\pi]v - p - v_0$	with prob. $(1-\rho)s\pi$
$r_p\|\bar{s}$:	$\pi v - p - v_0$	with prob. $\rho(1-s)\psi_\eta F_\alpha(\bar{\alpha}_{\bar{s}})$
$r_p\|0$:	$(1-\sigma)\pi v - p - v_0$	with prob. $\rho s(1-\pi)\psi_\eta F_\alpha(\bar{\alpha}_s)$
$r_p\|v$:	$[\sigma + (1-\sigma)\pi]v - p - v_0$	with prob. $\rho s\pi\psi_\eta$

  $\Rightarrow$ Trivially, CS lower with endorsements

  $\Rightarrow$ Effect of ψ_η ambiguous (increases "misleading" & aligned endorsement)

Example: HML-Equilibrium

- $v = 6,\ v_0 = 2,\ p = 3,\ \kappa = 0.5,\ \pi = 0.8, \rho = 0.7,\ s = 0.2,\ \sigma = 0.8$
- $\alpha \sim U[0,1] \quad \sigma_n \sim U[\sigma, 1]$



- $\bar{\alpha}_{\bar{s}} \approx 0.71,\ \bar{\alpha}_s \approx 0.11 \ \Rightarrow\ \psi_\alpha \approx 0.19$
- $\bar{\sigma} \approx 0.86 \ \Rightarrow\ \psi_\eta \approx 0.69 \qquad \bar{\kappa}_H \approx 0.7$

An Interesting Observation: The Non-Parasocial Equilibrium

Non-Parasocial Equilibria

There exist HML-Equilibria for which $\bar{\sigma} \leq \sigma$. That is, all consumers follow paid recommendation, even in the absence of a parasocial component (i.e., for a degenerate σ_η-distribution).

- Usually accompanied by low $\bar{\alpha}$ $\Rightarrow$ high alignment probability ψ_α
- Collective reputation of endorsing influencers good enough
- Consumer surplus still lower than without endorsements

$\Rightarrow$ Non-Parasocial Equilibria not limited to HML-Equilibria

The (first) ML-Endorsement Equilibrium

- Two (mutually exclusive) equilibria classes dependent on $\sigma v - p \gtrless v_0$

  $\Rightarrow$ focus (first) on $\sigma v - p > v_0$ [genuine recommendation for M-types]

ML-Endorsement Equilibrium (Aligned Influencers)

For intermediate κ there exists an equilibrium with endorsements in which

(a) Only firm types $q = M, L$ decide to use endorsements at cost κ.

(b) There exist thresholds $\bar{\alpha}_s$ and $\bar{\alpha}_{\bar{s}}$ such that an influencer

  i. with $\alpha \leq \bar{\alpha}_{\bar{s}}$ endorses the product unseen.

  ii. with $\alpha \leq \bar{\alpha}_s$ endorses any product and $\alpha > \bar{\alpha}_s$ only $v_\iota = v$ products.

(c) All consumers follow genuine recommendations.

(d) Sufficiently parasocial consumers ($\sigma_\eta > \bar{\sigma}$) also follow endorsements.

Firm Decision

- Firm endorses if costs are sufficiently small $\kappa \leq (D_E^q - D_0^q)(p - c)$

  $\Rightarrow$ effective demand with (D_E^q) and without (D_0^q) endorsements depends on v_ι

- Denote share of parasocial customers $(\sigma_\eta > \bar{\sigma})$ by ψ_η

v_ι	D_0	D_E
$\vee$	s	$\rho(1-s)\psi_\eta F_\alpha(\bar{\alpha}_{\bar{s}}) - \rho s(1-\psi_\eta) + s$
0	0	$\rho(1-s)\psi_\eta F_\alpha(\bar{\alpha}_{\bar{s}}) + \rho s\psi_\eta F_\alpha(\bar{\alpha}_s)$

- Note that absolute gains $(D_E^q - D_0^q)$ decrease with q_ι (and hence with q)

  $\Rightarrow$ H-type firm not endorsing iff $\kappa > \rho[\psi_\eta(1-s)F_\alpha(\bar{\alpha}_{\bar{s}}) - (1-\psi_\eta)s](p-c)$

  $\Rightarrow$ others endorse iff

  $$\kappa \leq \bar{\kappa}_M \equiv \rho[\psi_\eta(1-s)F_\alpha(\bar{\alpha}_{\bar{s}}) - \pi(1-\psi_\eta)s + (1-\pi)\psi_\eta s F_\alpha(\bar{\alpha}_s)](p-c)$$

Consumer Decision

- Reminder: Consumers can differentiate paid from genuine recommendations
  $\Rightarrow$ paid ones can still be **aligned** (i.e., iff $v_\iota = v$) for high α $\Rightarrow$ denote prob. ψ_α
- Expected value from unaligned recommendations: $\frac{\pi(1-\pi)}{1-\pi^2} v = \frac{\pi}{1+\pi} v$
- Genuine recommendations always aligned (Benchmark) $\Rightarrow$ always followed
- Follow paid recommendation iff v_0 or p sufficiently small and

  $\mathbb{E}[v_\theta | r^p] = \psi_\alpha \sigma v + (1 - \psi_\alpha) \frac{\pi}{1+\pi} v \geq v_0 + p$

  $\Rightarrow$ satisfied for sufficiently parasocial consumers $\Rightarrow \sigma_\eta \geq \bar{\sigma} \equiv \frac{v_0+p}{\psi_\alpha v} - \frac{\pi(1-\psi_\alpha)}{(1+\pi)\psi_\alpha}$

  $\Rightarrow$ denote probability of following paid recommendations by ψ_η

Influencer Decision

- Recall influencer's utility: $u_{\mathcal{I}} = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[u_c(r) - v_0]$
- Three influencer types, accept endorsement iff
  - $\bar{s}: \quad (1-\alpha)\kappa + \alpha[\frac{\pi}{1+\pi}v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \frac{\pi}{1+\pi}v + p + v_0}$
  - $s|v: \ (1-\alpha)\kappa \geq 0$
    $\Rightarrow$ always true (interests perfectly aligned as long as $\sigma v - p \geq v_0$)
  - $s|0: (1-\alpha)\kappa + \alpha[(1-\sigma)\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_s \equiv \frac{\kappa}{\kappa - (1-\sigma)\pi v + p + v_0}$

$\Rightarrow$ aligned paid recommendation prob. $\psi_\alpha = \frac{\pi s(1-F_\alpha(\bar{\alpha}_s))}{s(\pi + F_\alpha(\bar{\alpha}_s)) + (1+\pi)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})}$

Comparative Statics

Influencer

- $\bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \frac{\pi}{1+\pi} v + p + v_0}$ $\qquad$ $\bar{\alpha}_s \equiv \frac{\kappa}{\kappa - (1-\sigma)\pi v + p + v_0}$

  $\Rightarrow$ incr. in κ, π, v decr. in p, v_0 $\qquad$ incr. in κ, π, v decr. in p, v_0, σ

- $\psi_\alpha = \frac{\pi s(1-F_\alpha(\bar{\alpha}_s))}{s(\pi + F_\alpha(\bar{\alpha}_s)) + (1+\pi)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})}$

  $\Rightarrow$ incr. in s, p, v_0, σ decr. in κ, v and $\bar{\alpha}$ ambiguous in π

Consumer

- $\bar{\sigma} \equiv \frac{v_0 + p}{\psi_\alpha v} - \frac{\pi(1-\psi_\alpha)}{(1+\pi)\psi_\alpha}$

  $\Rightarrow$ incr. in κ decr. in ψ_α, s, σ

Firm

- $\bar{\kappa}_H \equiv \rho[\psi_\eta(1-s)F_\alpha(\bar{\alpha}_{\bar{s}}) - (1-\psi_\eta)s](p-c)$

  $\Rightarrow$ incr. in ρ decr. in c

ML-Equilibrium vs HML-Equilibrium

Influencer

- Uninformed influencer less likely to endorse: $\bar{\alpha}_{\bar{s}}^{ML} < \bar{\alpha}_{\bar{s}}^{HML}$
- Informed influencer unchanged: $\bar{\alpha}_{s}^{ML} = \bar{\alpha}_{s}^{HML}$

Consumer

- More parasociality needed to follow endorsements: $\bar{\sigma}^{ML} > \bar{\sigma}^{HML}$
- Value of genuine recommendation increases

Firm

- Changes in demand and threshold only through $\bar{\alpha}, \bar{\sigma}$
- Anticipating only ML-endorsements reduces profitability: $\bar{\kappa}_{H}^{ML} < \bar{\kappa}_{H}^{HML}$

  $\Rightarrow$ since $\bar{\kappa}_{H}^{ML} < \bar{\kappa}_{H}^{HML}$ $\rightarrow$ multiplicity of HML/ML equilibrium

Consumer Surplus

- Same derivations for welfare and consumer surplus as before
- Compare net consumer surplus

r_g:	$[\sigma + (1-\sigma)\pi]v - p - v_0$	with prob. $(1-\rho)s\pi$
$r_p\|\bar{s}$:	$\frac{\pi}{1+\pi}v - p - v_0$	with prob. $\rho(1-s)\psi_\eta F_\alpha(\bar{\alpha}_{\bar{s}})$
$r_p\|0$:	$(1-\sigma)\pi v - p - v_0$	with prob. $\rho s(1-\pi)\psi_\eta F_\alpha(\bar{\alpha}_s)$
$r_p\|v$:	$\sigma v - p - v_0$	with prob. $\rho s\pi\psi_\eta$

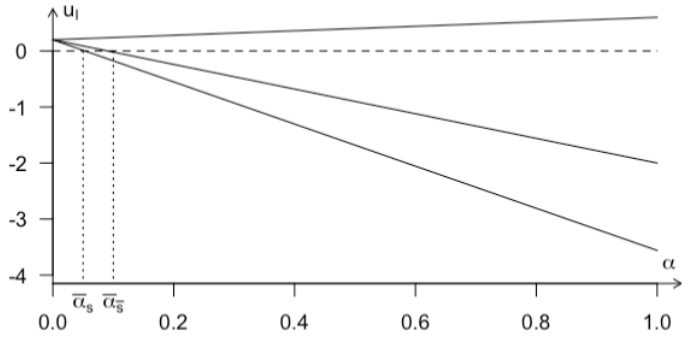
$\Rightarrow$ Again, CS lower with endorsements & effect of ψ_η ambiguous

$\Rightarrow$ Under multiplicity: $CS^{ML} \gtrless CS^{HML}$

- (Weakly) lower payoffs following paid recommendations [in blue]
- Probability of following paid recommendations decreases $[\bar{\alpha}_{\bar{s}} \downarrow, \psi_\eta \downarrow]$

Example: ML-Equilibrium

- $v = 8,\ v_0 = 1,\ p = 4,\ \kappa = 0.1,\ \pi = 0.6,\ \rho = 0.9,\ s = 0.3,\ \sigma = 0.7$
- $\alpha \sim U[0,1] \quad \sigma_\eta \sim U[\sigma, 1]$



- $\bar{\alpha}_{\bar{s}} \approx 0.1 \quad \bar{\alpha}_s \approx 0.05 \Rightarrow \psi_\alpha \approx 0.72$
- $\bar{\sigma} \approx 0.72 \Rightarrow \psi_\eta \approx 0.93 \qquad \bar{\kappa}_H \approx 0.04 \quad \bar{\kappa}_M \approx 0.11$

The other ML-Endorsement Equilibrium

- Two (mutually exclusive) equilibria classes dependent on $\sigma v - p \gtrless v_0$

  $\Rightarrow$ focus now on $\sigma v - p > v_0$ [some $v_\iota = v$ get not recommended]

ML-Endorsement Equilibrium (Non-Aligned Influencers)

For intermediate κ there exists an equilibrium with endorsements in which

(a) Only firm types $q = M, L$ decide to use endorsements at cost κ.

(b) There exist thresholds $\bar{\alpha}_0$, $\bar{\alpha}_v$ and $\bar{\alpha}_{\bar{s}}$ such that an influencer

  i. with $\alpha \leq \bar{\alpha}_{\bar{s}}$ endorses the product unseen.

  ii. with $\alpha \leq \bar{\alpha}_0$ endorses any product and $\alpha < \bar{\alpha}_v$ only $v_\iota = v$ products.

(c) All consumers follow genuine recommendations.

(d) Sufficiently parasocial consumers ($\sigma_\eta > \bar{\sigma}$) also follow endorsements.

Influencer Decision

- Similar to before, but with major difference at $v_\iota = v$

  $\Rightarrow$ additional info of endorsement offer now harmful

- Three influencer types, accept endorsement iff
  - $\bar{s}: \ \bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \frac{\pi}{1+\pi} v + p + v_0}$ [unchanged] $s|0: \ \bar{\alpha}_0 \equiv \frac{\kappa}{\kappa - (1-\sigma)\pi v + p + v_0}$ [unchanged]
  - $s|v: \ (1-\alpha)\kappa + \alpha[\sigma v - p - v_0] \geq 0$

    $\Rightarrow$ since $\sigma v - p < v_0$, now $\alpha < \bar{\alpha}_v \equiv \frac{\kappa}{\kappa - \sigma v + p + v_0}$

  $\Rightarrow$ One can show $\bar{\alpha}_v > \bar{\alpha}_0$ and $\bar{\alpha}_{\bar{s}} > \bar{\alpha}_0$

- Modified aligned paid recommendation probability $\psi_\alpha = \frac{s(F_\alpha(\bar{\alpha}_v) - F_\alpha(\bar{\alpha}_0))}{sF_\alpha(\bar{\alpha}_v) + (1-s)F_\alpha(\bar{\alpha}_{\bar{s}})}$

  $\Rightarrow$ Otherwise consumer problem unchanged: $\bar{\sigma} \equiv \frac{v_0 + p}{\psi_\alpha v} - \frac{\pi(1-\psi_\alpha)}{(1+\pi)\psi_\alpha}$

Firm Decision

- Firm endorses if costs are sufficiently small $\kappa \leq (D_E^q - D_0^q)(p - c)$
  $\Rightarrow$ effective demand with (D_E^q) and without (D_0^q) endorsements depends on v_ι

- Adjust endorsement demand D_E^q for abstaining influencer ($\alpha > \bar{\alpha}_v$)

v_ι	D_0	D_E
v	s	$\rho(1-s)\psi_\eta F_\alpha(\bar{\alpha}_{\bar{s}}) - \rho s(1-\psi_\eta)F_\alpha(\bar{\alpha}_v) - \rho s(1-F_\alpha(\bar{\alpha}_v)) + s$
0	0	$\rho(1-s)\psi_\eta F_\alpha(\bar{\alpha}_{\bar{s}}) + \rho s\psi_\eta F_\alpha(\bar{\alpha}_s)$

- Note that absolute gains ($D_E^q - D_0^q$) decrease with q_ι (and hence with q)
  $\Rightarrow$ adjust firm-type thresholds $\bar{\kappa}_H$ and $\bar{\kappa}_M$ accordingly

Non-Existence of L-Endorsement Equilibria

Non-Existence of L-Endorsement Equilibria

In the baseline, there cannot exist an equilibrium in which only the L-type uses endorsements while the other types rely solely on genuine recommendations.

- Equilibrium possible from firm side with sufficiently high κ
- Paid recommendations accepted by sufficiently selfish influencer (low $\bar{\alpha}$)
- paid recommendation credibly signals $q = L \Rightarrow v_\theta = 0$

  $\Rightarrow$ no paid recommendation followed by consumers

  $\Rightarrow$ no reason to endorse for L-type firm

A Sketch of the mL Mixed Endorsement Equilibrium

Mixed Endorsement Equilibria

In any equliibrium, at most one firm finds it optimal to mix at the same time.

- H-firm never endorses, L-firm always, M-firm with prob. ε

  $\Rightarrow$ expected value from endorsement: $\frac{\varepsilon\pi}{2\varepsilon\pi+1-\pi}v$

- Influencer decision: $\bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa-\frac{\varepsilon\pi}{2\varepsilon\pi+1-\pi}v+p+v_0}$, $\quad \bar{\alpha}_0 \equiv \frac{\kappa}{\kappa-(1-\sigma)\frac{\varepsilon\pi}{\varepsilon\pi+1-\pi}v+p+v_0}$

  $\Rightarrow$ aligned endorsement prob. : $\psi_\alpha = \frac{\varepsilon\pi s(1-F_\alpha(\bar{\alpha}_s))}{\varepsilon\pi s+(\varepsilon\pi+1-\pi)sF_\alpha(\bar{\alpha}_s)+(2\varepsilon\pi+1-\pi)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})}$

- Consumer decision: $\psi_\alpha\sigma + (1-\psi_\alpha)\frac{\varepsilon\pi}{2\varepsilon\pi+1-\pi} \geq \frac{v_0-p}{v} \quad \Rightarrow \bar{\sigma} \equiv \frac{v_0+p}{\psi_\alpha v} - \frac{\varepsilon\pi(1-\psi_\alpha)}{(2\varepsilon\pi+1-\pi)\psi_\alpha}$

- Firm decision: choose ε such that $\kappa = \bar{\kappa}_M(\varepsilon)$

Influencer Segmentation

- Influencers usually have a (publically observable) area of focus/expertise
  $\Rightarrow$ e.g. fashion, travel, tech, food, etc.

- Revisit product space:
  - Multiple quality dimensions **Used before**
    $\Rightarrow$ denote $j \in \{1, 2\}$
  - Multiple product categories **Now to be used**
    $\Rightarrow$ denote $k = S$ (specialization) or $k = G$ (general)

- Different behavior of influencer & consumer for $k = S$ vs. $k = G$
  (a) Influencers more aware of products in their area
  (b) Higher similarity to influencers in their area [$\rightarrow$ parasocial spillovers]

Influencer Segmentation – Improved Awareness

- Start from baseline model for general product $\rightarrow$ $s^G = s$
  $\Rightarrow$ in area of expertise $s^S > s$

- Aware influencers can give aligned recommendations: $\psi_\alpha^S > \psi_\alpha^G$
  $\Rightarrow$ value of paid recommendation increases

- Less skewed beliefs required $\bar{\sigma}^S < \bar{\sigma}^G$
  $\Rightarrow$ consumer more likely to follow paid recommendation $\psi_\eta^S > \psi_\eta^G$

- Equilibrium structure otherwise unchanged

Influencer Segmentation – Increased Similarity

- Increased match probability in area of expertise $\sigma^S > \sigma = \sigma^G$
- Polarizes $u_{\mathcal{I}}$ after knowing product: $\bar{\alpha}_s^S < \bar{\alpha}_s^G$ $\quad [\text{ also } \bar{\alpha}_v^S > \bar{\alpha}_v^G]$
  - $\Rightarrow$ non-aligned recommendations less likely: $\psi_\alpha^S > \psi_\alpha^G$
  - $\Rightarrow$ as before: less skewed beliefs required $\bar{\sigma}^S < \bar{\sigma}^G$ and $\psi_\eta^S > \psi_\eta^G$
  - $\Rightarrow$ Equilibrium structure otherwise unchanged

Alternative Parasocial Modeling – Similarity Spillover

- Increased but unskewed similarity in specialized product $\sigma_\eta^S = \sigma^S > \sigma$
- Spillover into general product $\sigma_\eta^G = \eta\sigma^S + (1-\eta)\sigma$

- Endorsement equilibria for specialized product only non-parasocial
- General product equilibria qualitatively unchanged

Generalizing the Influencer Decision

- Incorporate additional source of utility for influencer

  $\Rightarrow$ For easier tractability $\alpha \in [0,1] \rightarrow \alpha \in [0,\infty)$
- Utility takes form $u_{\mathcal{I}} = \Pi_{\mathcal{I}} + \alpha[u_c(r) - u_c(\emptyset)] - k(\rho)\mathbb{I}_{\{v_\theta | r=0\}} + Const.(\rho)$
- Example: Consumer-regarding preferences with monetized engagement
  - Each (future) period, earn m from engagement of followers (mass ρ)
  - Engagement ε reduces to $\underline{\varepsilon}$ following bad recommendation for τ periods

    $\Rightarrow$ occurs with probability $\zeta \equiv \mathbb{P}[v_\theta = 0|r]$

$$
\begin{aligned}
u_{\mathcal{I}} &= \Pi_{\mathcal{I}} + \Sigma_0^\infty \delta^t m\varepsilon(v)\rho + \alpha[u_c(r) - u_c(\emptyset)] \\
&= \Pi_{\mathcal{I}} + \Sigma_0^\tau \delta^t m[(1-\zeta)\varepsilon + \zeta\underline{\varepsilon}]\rho + \Sigma_{\tau+1}^\infty \delta^t m\varepsilon\rho + \alpha[u_c(r) - u_c(\emptyset)] \\
&= \Pi_{\mathcal{I}} + \underbrace{\Sigma_0^\tau \delta^t m\zeta\Delta\varepsilon\rho}_{=\frac{1-\delta^{\tau+1}}{1-\delta} m\zeta\Delta\varepsilon\rho \equiv k_\varepsilon(\rho)\zeta} + \underbrace{\Sigma_0^\infty \delta^t m\varepsilon\rho}_{=Const.(\rho)} + \alpha[u_c(r) - u_c(\emptyset)]
\end{aligned}
$$

Generalizing the Influencer Decision – Thresholds

- Revisiting the Equilibrium Thresholds derived from $u_{\mathcal{I}}$
- In HML-Equilibrium
  - $\bar{s}: \quad \kappa + \alpha[\pi v - p - v_0] - k_\varepsilon(\rho)(1-\pi) \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_{\bar{s}} \equiv \frac{\kappa - k_\varepsilon(\rho)(1-\pi)}{v_0 + p - \pi v}$
  - $s|v: \ \kappa + \alpha[(1-\pi)\sigma v + \pi v - p - v_0] - k_\varepsilon(\rho)(1-\sigma)(1-\pi) \geq 0$
    $\Rightarrow$ always true for sufficiently low k_ε [otherwise $\alpha \geq \bar{\alpha}_v$]
  - $s|0: \kappa + \alpha[(1-\sigma)\pi v - p - v_0] - k_\varepsilon(\rho)(1-\pi+\sigma\pi) \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_s \equiv \frac{\kappa - k_\varepsilon(\rho)(1-\pi+\sigma\pi)}{v_0 + p - (1-\sigma)\pi v}$

  $\Rightarrow$ Thresholds $\bar{\alpha}$ decreasing in ρ
- Results (otherwise) qualitatively unchanged [same for ML-equilibria]

Costly Quality Assessment

- Allow unaware influencers ($\bar{s}$) to learn v_ι ex-post at cost k_Q
  - $\Rightarrow$ no changes to influencer decision after $s|0$ and $s|v$
  - $\Rightarrow$ after investigating quality $\rightarrow$ only recommend if $v_\iota = v$
- Trade-off: higher consumer utility $\leftrightarrow$ less likely payment
  - $\Rightarrow$ assess quality iff $\alpha > \bar{\alpha}^Q_{\bar{s}} \equiv \frac{\kappa + k_Q(1-\pi)^{-1}}{\kappa - (1-\sigma)\pi v + p + v_0}$ [HML-Eq. case]
  - $\Rightarrow$ two possible cases dependent on $\bar{\alpha}^Q_{\bar{s}} \gtrless \bar{\alpha}_{\bar{s}}$
    - Unconditional acceptance (low α) or acceptance after positive assessment (high α)
    - additional unconditional rejection (intermediate α)
  - $\Rightarrow$ more aligned recommendations: $\psi_\alpha \uparrow$, $\bar{\sigma} \downarrow$, $\psi_\eta \uparrow$
- Qualitatively similar results for other equilibrium clases

What's ahead

- Influencer indirect utility conditional on following recommendation (status quo)?
  $\Rightarrow$ endorsements crowd out genuine recommendations
- Recommendation confounding (paid appears genuine)
  $\Rightarrow$ Following genuine recommendation may require parasociality
- Endogeneizing κ
- Competition between firms (and/or influencers)
- Two-stage price setting model

That's All Folks!

If you have questions, comments or feedback,
don't hesitate to contact me or stop by.

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Appendix: Influencer Decision with unconditional utility

- Recall influencer's utility: $u_{\mathcal{I}} = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[u_c(r) - v_0]$
- Three influencer types, accept endorsement iff
  - $\bar{s}: \quad (1-\alpha)\kappa + \alpha\psi_\eta[\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \psi_\eta(\pi v - p - v_0)}$
  - $s|v: \ (1-\alpha)\kappa + \alpha\psi_\eta[\sigma v + (1-\sigma)\pi v - p - v_0] \geq \alpha[\sigma v + (1-\sigma)\pi v - p - v_0]$
    $\Rightarrow \alpha \leq \bar{\alpha}^v_{\bar{s}} \equiv \frac{\kappa}{\kappa + (1-\psi_\eta)(\sigma v + (1-\sigma)\pi v - p - v_0)}$
  - $s|0: (1-\alpha)\kappa + \alpha\psi_\eta[(1-\sigma)\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}^0_s \equiv \frac{\kappa}{\kappa - \psi_\eta((1-\sigma)\pi v - p - v_0)}$

$\Rightarrow$ one can show: $\bar{\alpha}^v_{\bar{s}} > \bar{\alpha}_{\bar{s}} > \bar{\alpha}^0_s$