Advertising, Social Media and Parasocial Recommendations

New Infuencer-Centered Modeling Approach

Julian Klix | Preliminary Draft | May, 2026

UNIVERSITÄT MANNHEIM

Model Setup: The Product Landscape

- Multiple product categories

  ⇒ denote $k \in \{A, B, \ldots\}$

  ⇒ will allow for influencer "specialization"

  ⇒ relevant only for informational purposes

  Not yet used right now

- Multiple quality dimensions

  ⇒ denote $j \in \{1, 2, \ldots\}$

  ⇒ relevant for realized utility

Model Setup: The Firm Side

The Monopolist

- Single firm with high (H) or low (L) quality in each dimension [generically q_j]
- Realization of all q_j are private information with $P[q_j = H] \equiv \pi$
- Produced at constant, homogeneous marginal cost c
- Competes (indirectly) against outside option
- Can advertise product on social media at exogenous cost κ

The Outside Option

- Available instead of purchasing the product

$\Rightarrow$ Can be thought of as background Bertrand competition with cost $c_0 = v_0 + p_0$

Model Setup: The Consumer Side

- One (representative) consumer with unit demand
- Randomly drawn product dimension type $\theta \in J$
- Receives utility v_0 from outside option and v_q from the new product
  $\Rightarrow$ utility of product given by $u_c(\theta) = v_{q_\theta}$ [i.e. quality of dimension θ]
  $\Rightarrow$ normalize $v_L = 0$, simplify notation $v_H = v$

- Can obtain **one** recommendation r prior to purchase
  $\Rightarrow$ either from environment (non-strategic) or social media (potent. strategic)

Assumption 1

The outside option v_0 is sufficiently large to dominate random purchase.
That is, $v_0 > \pi v - p$.

Model Setup: The Influencer Side

- One (representative) Inluencer that can give recommendations
  - Product dimension type $\iota \in J$
  - direct utility from payment, indirect utility from value-added to consumer

    $\Rightarrow$ altruism parameter α captures trade-off, distr. following (cont.) F_α

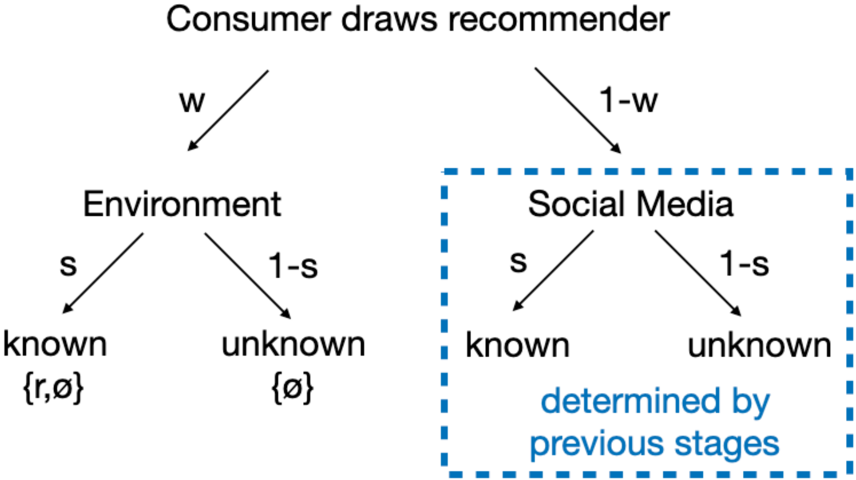
    $$u_{\mathcal{I}} = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[u_c(r) - u_c(\emptyset)] = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[\mathbb{E}[v_\theta|r] - p - v_0]$$

  $\Rightarrow$ non-strategic environment encapsuled by $\alpha = 1$
- Type-match probability (“similarity”) σ $\Rightarrow$ potentially σ_0, σ_I

Parasocial Relationships

The consumers parasocial relationship is captured by overestimating the influencer’s similarity, i.e., $\sigma_\eta > \sigma$. [Specific form to be determined]

Model Setup: The Recommendation Procedure



Model Setup: Timing

1. Fundamentals are drawn $(p, \kappa, q, v_j, \ldots)$
2. Firms learn q, knowing recommenders learn v_ι
3. Firm decides on endorsement
4. Influencer decides on accepting endorsements
5. Consumer draws recommender
6. Consumer observers potential recommendation
   $\Rightarrow$ can distinguish genuine from paid recommendations
7. Consumer makes purchase decision

$\Rightarrow$ utilities are realized

An Endorsement-Free Benchmark

- Outside option of v_0 $\Rightarrow$ by assumption, obtained without recommendation
- Consumer updates belief about v_θ after receiving a recommendation: $\pi \xrightarrow{r} \pi_r$

$$\pi_r \equiv \mathbb{P}[v_\theta = v|r] = \sigma \frac{1 \cdot \mathbb{P}[v]}{1 \cdot \mathbb{P}[v] + 0 \cdot \mathbb{P}[0]} + (1-\sigma) \frac{\pi \cdot \mathbb{P}[v]}{\pi \cdot \mathbb{P}[v] + \pi \cdot \mathbb{P}[0]} = \sigma + (1-\sigma)\pi$$

$\Rightarrow$ assume expected utility $\pi_r v - p = \pi v + (1-\pi)\sigma v - p > v_0$

- without endorsements ($\Pi_{\mathcal{I}} = 0$), influencer's interests perfectly aligned

$\Rightarrow$ recommend a (known) product iff $v_\iota = v$

Equilibrium of the Endorsement-Free Benchmark

Without endorsements, there is a unique equilibrium in which any high-value product gets recommended and all recommendations are followed.

The HML-Endorsement Equilibrium

HML-Endorsement Equilibrium

For sufficiently small κ there exists an equilibrium with endorsements in which

(a) All firm types ($q = H, M, L$) decide to use endorsements at cost κ.

(b) There exist thresholds $\bar{\alpha}_s$ and $\bar{\alpha}_{\bar{s}}$ such that an influencer

i. with $\alpha \leq \bar{\alpha}_s$ endorses the product unseen.

ii. with $\alpha \leq \bar{\alpha}_s$ endorses any product and $\alpha > \bar{\alpha}_{\bar{s}}$ only $v_\iota = v$ products.

(c) All consumers follow genuine recommendations.

(d) Sufficiently parasocial consumers ($\sigma_\eta > \bar{\sigma}$) also follow endorsements.

- Equilibrium with full firm-type pooling

Firm Decision

- Firm endorses if costs are sufficiently small

$$\kappa \leq (D_0^q - D_E^q)(p - c)$$

$\Rightarrow$ effective demand with (D_E^q) and without (D_0^q) endorsements depends on v_ι

	$v_\iota = v$	$v_\iota = 0$
D_0	s	0
D_E	$s + (1-\omega)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})$	$(1-\omega)sF_\alpha(\bar{\alpha}_s) + (1-\omega)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})$

- Note that absolute gains ($D_0^q - D_E^q$) decrease with q

$\Rightarrow$ all firms use endorsements as long as $\kappa \leq (1-\omega)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})(p-c)$

Consumer Decision

- Reminder: Consumers can differentiate paid from genuine recommendations
  $\Rightarrow$ paid ones can still be **aligned** (i.e., iff $v_\iota = v$) for high α $\Rightarrow$ denote prob. ψ_α

- Genuine recommendations always aligned (Benchmark) $\Rightarrow$ always followed

- Follow paid recommendation iff v_0 or p sufficiently small and

$$\mathbb{E}[v_\theta | r^p] = \psi_\alpha(\pi + (1-\pi)\sigma)v + (1-\psi_\alpha)\pi v = [\pi + (1-\pi)\sigma\psi_\alpha]v \geq v_0 + p$$

$\Rightarrow$ satisfied for sufficiently parasocial consumers $\Rightarrow \sigma \geq \frac{v_0+p}{(1-\pi)\psi_\alpha v} - \frac{\pi}{(1-\pi)\psi_\alpha}$

$\Rightarrow$ denote probability of following paid recommendations by ψ_η

Influencer Decision

- Recall influencer's utility: $u_{\mathcal{I}} = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[u_c(r) - v_0]$
- Three influencer types, accept endorsement iff
  - $\bar{s}: \quad (1-\alpha)\kappa + \alpha[\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \pi v + p + v_0}$
  - $s|v: \ (1-\alpha)\kappa \geq 0$
    $\Rightarrow$ always true (interests perfectly aligned)
  - $s|0: (1-\alpha)\kappa + \alpha[(1-\sigma)\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_s \equiv \frac{\kappa}{\kappa - (1-\sigma)\pi v + p + v_0}$

$\Rightarrow$ aligned paid recommendation probability $\psi_\alpha = \frac{s(1-F_\alpha(\bar{\alpha}_s))}{s+(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})}$

The (first) ML-Endorsement Equilibrium

- Two (mutually exclusive) equilibria classes dependent on $\sigma v - p \gtrless v_0$

  $\Rightarrow$ focus (first) on $\sigma v - p > v_0$ [genuine recommendation for M-types]

ML-Endorsement Equilibrium (Aligned Influencers)

For intermediate κ there exists an equilibrium with endorsements in which

(a) Only firm types $q = M, L$ decide to use endorsements at cost κ.

(b) There exist thresholds $\bar{\alpha}_s$ and $\bar{\alpha}_{\bar{s}}$ such that an influencer

  i. with $\alpha \leq \bar{\alpha}_s$ endorses the product unseen.

  ii. with $\alpha \leq \bar{\alpha}_s$ endorses any product and $\alpha > \bar{\alpha}_{\bar{s}}$ only $v_\iota = v$ products.

(c) All consumers follow genuine recommendations.

(d) Sufficiently parasocial consumers ($\sigma_\eta > \bar{\sigma}$) also follow endorsements.

Firm Decision

- Firm endorses if costs are sufficiently small

$$\kappa \leq (D_0^q - D_E^q)(p - c)$$

$\Rightarrow$ effective demand with (D_E^q) and without (D_0^q) endorsements depends on v_ι

	$v_\iota = v$	$v_\iota = 0$
D_0	s	0
D_E	$s + (1-\omega)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})$	$(1-\omega)sF_\alpha(\bar{\alpha}_s) + (1-\omega)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})$

- Note that absolute gains ($D_0^q - D_E^q$) decrease with q

$\Rightarrow$ H-type firm not endorsing as long as $\kappa > (1-\omega)(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})(p-c)$

- Changes in demand and cost thresolds only through changes in $\bar{\alpha}$

Consumer Decision

- Reminder: Consumers can differentiate paid from genuine recommendations
  $\Rightarrow$ paid ones can still be **aligned** (i.e., iff $v_\iota = v$) for high α $\Rightarrow$ denote prob. ψ_α
- Genuine recommendations always aligned (Benchmark) $\Rightarrow$ always followed
- Follow paid recommendation iff v_0 or p sufficiently small and

$$\mathbb{E}[v_\theta | r^p] = \psi_\alpha \sigma v + (1 - \psi_\alpha)\frac{\pi}{1+\pi} v \geq v_0 + p$$

$\Rightarrow$ satisfied for sufficiently parasocial consumers $\Rightarrow \sigma \geq \frac{v_0+p}{\psi_\alpha v} - \frac{\pi(1-\psi_\alpha)}{(1-\pi)\psi_\alpha}$

$\Rightarrow$ denote probability of following paid recommendations by ψ_η

Influencer Decision

- Recall influencer's utility: $u_{\mathcal{I}} = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[u_c(r) - v_0]$
- Three influencer types, accept endorsement iff
  - $\bar{s}: \quad (1-\alpha)\kappa + \alpha[\frac{\pi}{1+\pi}v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \frac{\pi}{1+\pi}v + p + v_0}$
  - $s|v: \ (1-\alpha)\kappa \geq 0$
    $\Rightarrow$ always true (interests perfectly aligned as long as $\sigma v - p \geq v_0$)
  - $s|0: (1-\alpha)\kappa + \alpha[(1-\sigma)\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_s \equiv \frac{\kappa}{\kappa - (1-\sigma)\pi v + p + v_0}$

$\Rightarrow$ aligned paid recommendation probability $\psi_\alpha = \frac{s(1-F_\alpha(\bar{\alpha}_s))}{s+(1-s)F_\alpha(\bar{\alpha}_{\bar{s}})}$

ML-Equilibrium vs HML-Equilibrium

Influencer

- Uninformed influencer less likely to endorse: $\bar{\alpha}_{\bar{s}}^{ML} < \bar{\alpha}_{\bar{s}}^{HML}$
- Informed influencer unchanged: $\bar{\alpha}_{s}^{ML} < \bar{\alpha}_{s}^{HML}$

Consumer

- To Show: More parasociality needed to follow endorsements: $\bar{\sigma}^{ML} > \bar{\sigma}^{HML}$
- Value of genuine recommendation increases

Firm

- Changes in demand and threshold only through $\bar{\alpha}$

$\Rightarrow$ Since $\bar{\kappa}_H^{HML} > \bar{\kappa}_H^{ML}$ $\rightarrow$ multiplicity of HML/ML equilibrium

Non-Existence of L-Endorsement Equilibria

Non-Existence of L-Endorsement Equilibria

In the baseline, there cannot exist an equilibrium in which only the L-type uses endorsements while the other types rely solely on genuine recommendations.

- Equilibrium possible from firm side with sufficiently high κ
- Paid recommendations accepted by sufficiently selfish influencer (low $\bar{\alpha}$)
- paid recommendation credibly signals $q = L \quad \Rightarrow v_\theta = 0$

$\Rightarrow$ no paid recommendation followed by consumers

$\Rightarrow$ no reason to endorse for L-type firm

What's ahead

- Influencer indirect utility conditional on following recommendation (status quo)?
- Continuous vs. binary parasocial state space?
- Characterize the prob. 0 mixed endorsement equilibria?

- ML-Endorsement Equilibrium
- Comparative Statics and Welfare/CS Analysis

- Introduce product categories and influencer "specialization" $\zeta \in K$

  $\Rightarrow$ affects σ and possibly ω_ζ $\rightarrow$ allows parasociality as spillover effects
- Endogenize ω (firm choice) $\rightarrow$ limited signalling
- Costly quality investigation for unknowing influencer

That’s All Folks!

If you have questions, comments or feedback,
don't hesitate to contact me or stop by.

julian.klix@uni-mannheim.de

L7 3-5, Room 3.43

UNIVERSITÄT MANNHEIM

Appendix: Influencer Decision with unconditional utility

- Recall influencer's utility: $u_{\mathcal{I}} = (1-\alpha)\Pi_{\mathcal{I}} + \alpha[u_c(r) - v_0]$
- Three influencer types, accept endorsement iff
  - $\bar{s}: \quad (1-\alpha)\kappa + \alpha\psi_\eta[\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_{\bar{s}} \equiv \frac{\kappa}{\kappa - \psi_\eta(\pi v - p - v_0)}$
  - $s|v: \ (1-\alpha)\kappa + \alpha\psi_\eta[\sigma v + (1-\sigma)\pi v - p - v_0] \geq \alpha[\sigma v + (1-\sigma)\pi v - p - v_0]$
    $\Rightarrow \alpha \leq \bar{\alpha}_{\bar{s}}^{v} \equiv \frac{\kappa}{\kappa + (1-\psi_\eta)(\sigma v + (1-\sigma)\pi v - p - v_0)}$
  - $s|0: (1-\alpha)\kappa + \alpha\psi_\eta[(1-\sigma)\pi v - p - v_0] \geq 0$
    $\Rightarrow \alpha \leq \bar{\alpha}_{s}^{0} \equiv \frac{\kappa}{\kappa - \psi_\eta((1-\sigma)\pi v - p - v_0)}$

$\Rightarrow$ one can show: $\bar{\alpha}_{\bar{s}}^{v} > \bar{\alpha}_{\bar{s}} > \bar{\alpha}_{s}^{0}$